APAB – ch 3 HW review

3.1 #17, 21, 33, 37, 45
3.2 # 7, 13, 34, 47, 49
3.3 # 13, 37, 41, 45, 49
3.4 # 50, 51, 52, 54, 56
3.5 # 5, 17

DONE IN CLASS
3.6 # 13, 33
3.7 # 15, 27

3.1

(#17) $h(x) = \sin^2 x + \cos x$ $(0, 2\pi)$

$h'(x) = 2\sin x \cos x - \sin x$

CRIT. PTS

$h' = 0 = 2\sin x \cos x - \sin x$

$0 = \sin x (2\cos x - 1)$

$\sin x = 0$ $\quad x = \pi$

$2\cos x - 1 = 0$

$\cos x = 1/2$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

h' DNE never

included end pts none

ANS: $x = \pi/3, 5\pi/3, \pi$

(21) $f(x) = -x^2 + 3x$ $[0, 3]$

$f'(x) = -2x + 3$

CRIT PTS

$f' = 0$

$0 = -2x + 3$

$x = 3/2$

f' DNE never

included end pts $x = 0, 3$

	0	3/2	3
$-2x+3$	+	0	−
F'	+		−
		MAX	

MIN MIN

ABS MAX $(3/2, \underline{9/4})$ ← $f(3/2) = -(\frac{9}{4}) + \frac{9}{2} = -\frac{9}{4} + \frac{18}{4} = 9/4$

ABS MIN $(3, \underline{0})$ ← $f(3) = -9 + 9$

ABS MIN $(0, \underline{0})$ ← $f(0) = 0 + 0 = 0$

↑ since both y-vals are the same; they are both the absolute min

APAB Hw Review Ch 3

(33) $f(x) = \cos \pi x \quad [0, 1/6]$

$f'(x) = -\sin \pi x \cdot \pi = -\pi \sin \pi x$

Crit pts

$f' = 0 = -\pi \sin \pi x$

$0 = -\pi$ lie

$0 = \sin \pi x$

$\frac{\pi x}{\pi} = \frac{0, \pi, 2\pi, \ldots}{\pi}$

$x = 0, 1, 2 \ldots$

only $x = 0$ is in interval

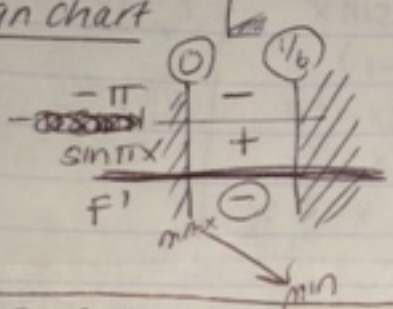
f' DNE never (sine is continuous) D: $(-\infty, \infty)$

included end pts

$x = 0$

$x = 1/6$

Sign chart

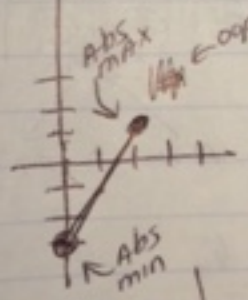


Abs max $(0, 1)$ ← $\cos \pi(0) = \cos 0 = 1$

Abs min $\left(\frac{1}{6}, \frac{\sqrt{3}}{2}\right)$ $\cos \frac{1}{6}\pi = \cos \frac{\pi}{6} = \sqrt{3}/2$

(37) $f(x) = 2x - 3$ is a line with ⊕ slope

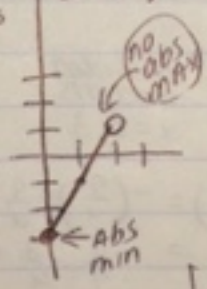
Ⓐ $[0, 2]$



Abs max $(2, 1)$

Abs min $(0, -3)$

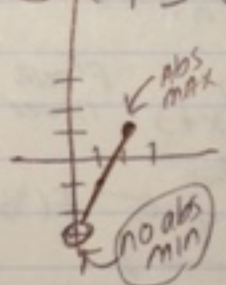
Ⓑ $[0, 2)$



Abs max none

Abs min $(0, -3)$

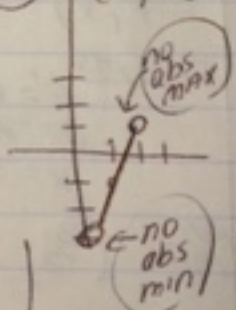
Ⓒ $(0, 2]$



Abs max $(2, 1)$

abs min none

Ⓓ $(0, 2)$



Abs max none

Abs min none

APAB HW review – Ch. 3

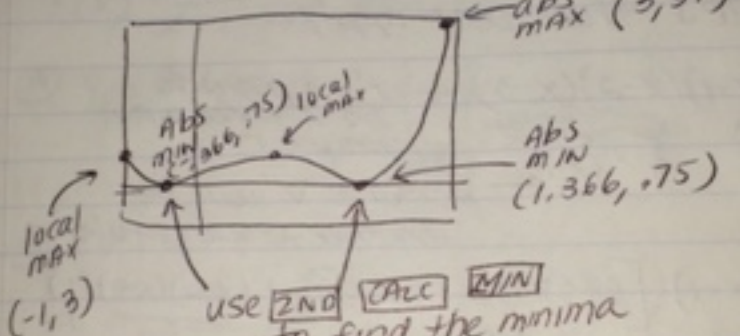
3.1 #45 (this is a calculator problem)

$f(x) = x^4 - 2x^3 + x + 1$ $[-1, 3]$ ← because of the

$Y_1 = X\text{^}4 - 2X\text{^}3 + X + 1$

$f(-1) = (-1)^4 - 2(-1)^3 + (-1) + 1$
$= 1 + 2 - 1 + 1 = 3$

$f(3) = 3^4 - 2(3)^3 + 3 + 1 = 81 - 2(27) + 4$
$= 81 - 54 + 4 = 81 - 50 = 31$



3.2 # 7, 13, 34, 47, 49

$f(x) = x^{u}\sqrt{x+4}^{v}$

⑦ $f(x) = x\sqrt{x+4}$

$f'(x) = x\frac{1}{2\sqrt{x+4}}(1) + \sqrt{x+4}(1)$

X-INT (Y=0)

$0 = x\sqrt{x+4}$

$x = 0$ $\quad 0 = \sqrt{x+4}$

$0 = x + 4$

$-4 = x$

$f'(x) = \frac{x}{2\sqrt{x+4}} + \sqrt{x+4}$

$0 = \frac{x}{2\sqrt{x+4}} + \sqrt{x+4}$

$-\sqrt{x+4} = \frac{x}{2\sqrt{x+4}}$

$-2\sqrt{x+4}^2 = x$

$-2(x+4) = x$

$-2x - 8 = x$

$-8 = 3x$

$-8/3 = x$

since x is $-4 < x < 0$

$x + 4 = +$

$\sqrt{x+4}^2 = x + 4$

so $f'(-8/3) = 0$

AND

$-4 < -8/3 < 0$

APAB HW review

(13) $f(x) = (x-1)(x-2)(x-3)$ $[1,3]$

For Rolle's thm to apply...

✓ (1) $f(x)$ is continuous – yes, it's a polynomial

✓ (2) $f(x)$ is differentiable – yes, it's a polynomial

✓ (3) $f(b) = f(a)$ ... $f(1) = 0(-1)(-2) = 0$ true

$f(3) = 2(1)(0) = 0$

So Rolle's thm does apply...

$f(x) = (x-1)(x-2)(x-3)$ ← it's again to do a three way product...

u ↑ v ← because "v" will be another product rule

$f'(x) = (x-1)\underbrace{[(x-2)(1) + (x-3)(1)]}_{dv} + \underbrace{(x-2)(x-3)}_{v}(1)$ du

$f'(x) = (x-1)(x-2) + (x-1)(x-3) + (x-2)(x-3)$

No GCF... so FOIL it...

$= x^2 - 3x + 2 + x^2 - 4x + 3 + x^2 - 5x + 6$

$f'(x) = 3x^2 - 12x + 11$ ← instant slope

Avg slope $= \frac{f(3) - f(1)}{3-1} = \frac{0-0}{3-1} = \frac{0}{2} = 0$

instant slope = avg slope

$f'(x) = 3x^2 - 12x + 11 = 0$

$(3x \quad)(x \quad)$ 1 and 11... nope, won't factor

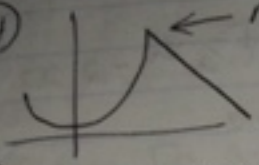
Use quad form $x = \frac{+12 \pm \sqrt{144 - 4(3)(11)}}{6} = \frac{144 - 132}{6} = \frac{12 \pm \sqrt{12}}{6} = \frac{12 \pm 2\sqrt{3}}{6}$

$\boxed{x = \frac{6 \pm \sqrt{3}}{3}}$ ← both are in $[1,3]$

APAB HW review ch 3

3.2 #34, 47, 49

(34)

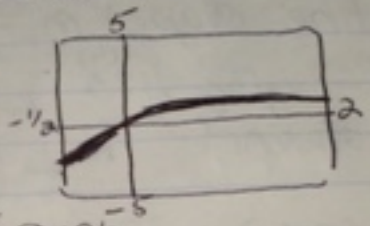


← MVT doesn't apply because f(x) is not differentiable

(47) this is A CALCULATOR problem because of the [A] symbol

$f(x) = \frac{x}{x+1}$ $\quad [-\frac{1}{2}, 2]$

(A) $Y_1 = X/(X+1)$



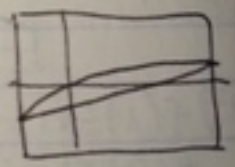
(B) SECANT line through $(-\frac{1}{2}, \frac{-1}{\frac{-\frac{1}{2}}{-\frac{1}{2}+1}})$ and $(2, \frac{2/3}{\uparrow 2/2+1 = 2/3})$

$\frac{-\frac{1}{2}}{\frac{1}{2}} = -1$

$$m = \frac{2/3 - (-1)}{2 - (-1/2)} = \frac{\frac{2}{3} + \frac{3}{3}}{\frac{4}{2} + \frac{1}{2}} = \frac{\frac{5}{3}}{\frac{5}{2}} = \frac{5}{3} \cdot \frac{2}{5} = \frac{2}{3}$$

$Y + 1 = \frac{2}{3}(X + 1/2)$

$Y_2 = (2/3)(X + (1/2)) - 1$



(C) $f'(x) = \frac{(x+1)(1) - x(1)}{(x+1)^2} = \frac{x+1-x}{(x+1)^2} = \frac{1}{(x+1)^2}$

$f'(x) = \frac{2}{3}$

$\frac{1}{(x+1)^2} = \frac{2}{3}$

$3 = 2(x+1)^2$

$\frac{3}{2} = \frac{2}{2}(x+1)^2$

$\frac{3}{2} = (x+1)^2$

$\pm\sqrt{\frac{3}{2}} = x+1$

$-1 \pm \sqrt{\frac{3}{2}} = x$

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3.2 #47 (cont.)

$x = -1 \pm \sqrt{\frac{3}{2}}$

$x = -1 + \sqrt{3/2} \approx .224$ (S)

$x = -1 - \sqrt{3/2} \approx -2.224$ (S)

NOTE: -2.224 (S) is NOT IN $[-1/2, 2]$ interval...

so only $x = -1 + \sqrt{3/2} \approx .224$ (S) is the answer.

tangent line slope $m_T = 2/3$

POINT $(-1+\sqrt{3/2}, .183)$ (4)

STO→A

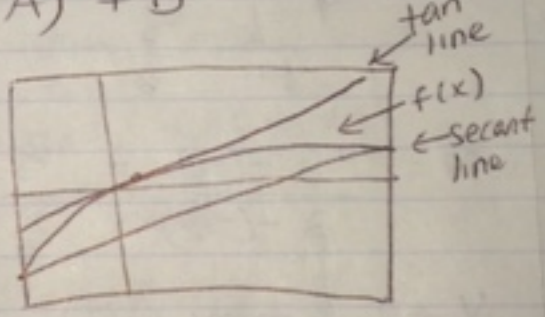
← plug into fxn

≈ .183503419 1

STO→B

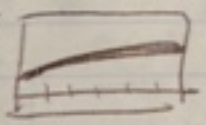
$Y_3 - B = (2/3)(X - A)$

$Y_3 = (2/3)(X - A) + B$



(49) $f(x) = \sqrt{x}$ $[1, 9]$

(A) $Y_1 = \sqrt{X}$



(B) $f(1) = \sqrt{1} = 1$ $(1, 1)$

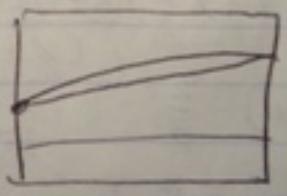
$f(9) = \sqrt{9} = 3$ $(9, 3)$

Secant line

$m = \frac{3-1}{9-1} = \frac{2}{8} = \frac{1}{4}$

$Y - 1 = \frac{1}{4}(X - 1)$

$\boxed{Y_2 = (1/4)(X-1) + 1}$



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3.2 #49 cont

(c) $f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$ → $\frac{1}{2\sqrt{x}} = \frac{1}{4}$

$2\sqrt{x} = 4$

$\sqrt{x} = 2$

$x = 4$

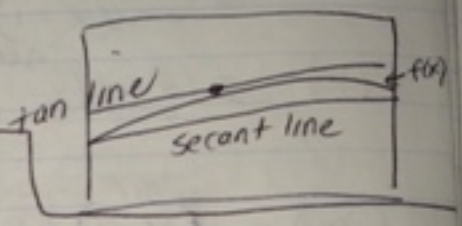
$f(4) = \sqrt{4} = 2$

$(4, 2)$

tangent line

$y - 2 = \frac{1}{4}(x - 4)$

$\boxed{Y_3 = (1/4)(x-4)+2}$



3.3 #13, 37, 41, 45, 49

3.4 #50, 51, 52, 54, 56

3.5 #5, 17

(13) $y = x\sqrt{16-x^2}$

$u \quad v$

$y' = x\frac{1}{2\sqrt{16-x^2}}(-2x) + \sqrt{16-x^2}(1) = \frac{-x^2}{\sqrt{16-x^2}} + \frac{\sqrt{16-x^2}}{1}$

$y' = \frac{-x^2}{\sqrt{16-x^2}} + \frac{\sqrt{16-x^2}}{1}\left(\frac{\sqrt{16-x^2}}{\sqrt{16-x^2}}\right)$

$y' = \frac{-x^2 + (16-x^2)}{\sqrt{16-x^2}} = \frac{-2x^2+16}{\sqrt{16-x^2}} = y'$

CRIT PTS

$y' = 0 = -2x^2 + 16$

$0 = -2(x^2 - 8)$

$0 = -2$ lie

$x^2 - 8 = 0$

$x = \pm\sqrt{8}$

y' DNE

$\sqrt{16-x^2} = 0$

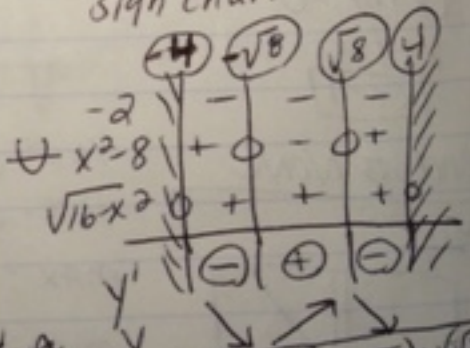
$16 - x^2 = 0$

$x^2 = 16$

$x = \pm 4$

also the endpts of domain of original fxn

sign chart



Dec: $(-4, -\sqrt{8}) \cup (\sqrt{8}, 4)$

Inc: $(-\sqrt{8}, \sqrt{8})$

APAB – ch.3 HW review

3.3 # 37, 41, 45, 49
3.4 # 50, 51, 52, 54, 56
3.5 # 5, 17

(37) $f(x) = \frac{x^2-2x+1}{(x+1)}$ ← VA @ $x=-1$

$$f'(x) = \frac{(x+1)(2x-2)-(x^2-2x+1)(1)}{(x+1)^2}$$

$$f'(x) = \frac{2(x+1)\overset{\text{GCF}}{(x-1)} - (x-1)\overset{\text{GCF}}{(x-1)}}{(x+1)^2}$$

$$f'(x) = \frac{\overset{\text{GCF}}{(x-1)}\left[2(x+1)-(x-1)\right]}{(x+1)^2}$$

$$f'(x) = \frac{(x-1)(2x+2-x+1)}{(x+1)^2} = \frac{(x-1)(x+3)}{(x+1)^2}$$

CRIT PTS

$f'(x) = 0 = (x-1)(x+3)$

$0 = (x-1)$ → $x = 1$

$0 = x+3$ → $-3 = x$

$f'(x)$ DNE

$(x+1)^2 = 0$

$x+1 = 0$

$x = -1$

Sign chart

	$-\infty$	-3	-1	1	∞
$x-1$	−	−	−	0	+
$x+3$	−	0	+	+	+
$\oplus$ → $(x+1)^2$	+	+	0	+	+
f'	+	−	−	+	
f	↑	max	↓	min	↑

INC: $(-\infty, -3) \cup (1, \infty)$

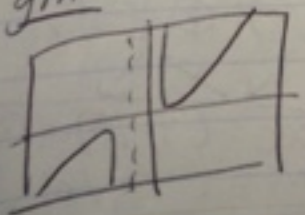
DEC: $(-3, -1) \cup (-1, 1)$

MAX: $(-3, \underline{-8})$ ← $f(-3) = \frac{9+6+1}{-2}$

MIN: $(1, \underline{0})$ ← $\frac{1-2+1}{2} = 0$

endpts: NONE

GRAPH



APAB ch 3 HW review

3.3 #41, 45, 49 3.5 #5, 17
3.4 #50, 51, 52, 54, 56

(41) $f(x) = \sin x + \cos x$ ON $(0, 2\pi)$

$f'(x) = \cos x - \sin x$

crit pts

$f'(x) = 0$

$0 = \cos x - \sin x$

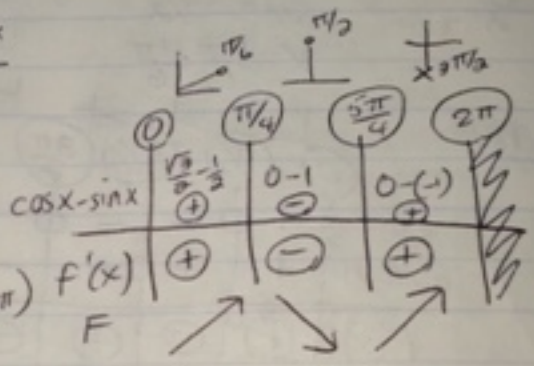
$\cos x = \sin x$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

f' DNE never

endpts $x = 0$, $x = 2\pi$] but NOT included

sign chart:

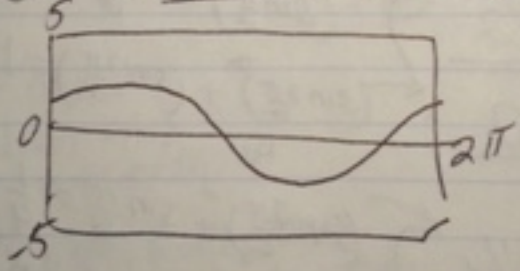


(A) INC $(0, \frac{\pi}{4}) \cup (\frac{5\pi}{4}, 2\pi)$
DEC $(\pi/4, 5\pi/4)$

(B) MAX $\left(\frac{\pi}{4}, \sqrt{2}\right)$ $\sin\frac{\pi}{4} + \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$

MIN $\left(\frac{5\pi}{4}, -\sqrt{2}\right)$ $\sin\frac{5\pi}{4} + \cos\frac{5\pi}{4} = \frac{-\sqrt{2}}{2} + \frac{-\sqrt{2}}{2}$

(C) (be in radian mode)



APAB - HW review Ch. 3

3.3 #45 $(0, 2\pi)$

$f(x) = \sin^2 x + \sin x$

$f'(x) = 2\sin x \cos x + \cos x$

$f'(x) = 0 = 2\sin x \cos x + \cos x$

crit pts $0 = \cos x (2\sin x + 1)$

$\cos x = 0$ | $2\sin x + 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ | $2\sin x = -1$

$\sin x = -1/2$

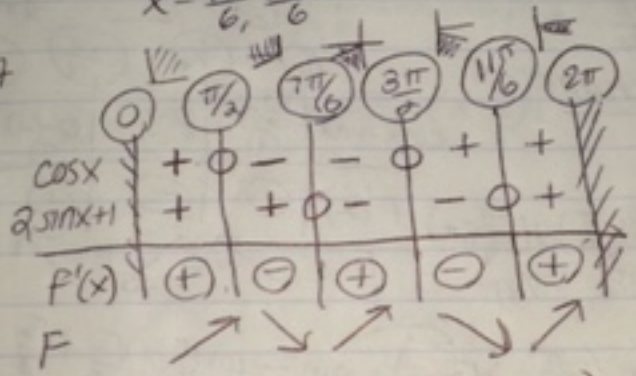
$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

F one nice endpts

$x = 0$

$x = 2\pi$ (NOT included)

Sign chart



(A) INC: $(0, \pi/2) \cup (\frac{7\pi}{6}, \frac{3\pi}{2}) \cup (\frac{11\pi}{6}, 2\pi)$

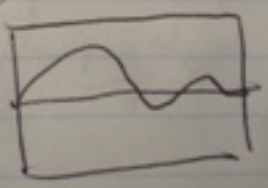
DEC: $(\frac{\pi}{2}, \frac{7\pi}{6}) \cup (\frac{3\pi}{2}, \frac{11\pi}{6})$

(B) Abs MAX: $(\frac{\pi}{2}, 2)$ $(\sin\frac{\pi}{2})^2 + \sin\frac{\pi}{2} = 1+1$

local MAX: $(\frac{3\pi}{2}, 0)$ $(\sin\frac{3\pi}{2})^2 + (\sin\frac{3\pi}{2}) = 1-1$

Abs MIN $(\frac{7\pi}{6}, -1/4)$ $(\sin\frac{7\pi}{6})^2 + \sin\frac{7\pi}{6} = \frac{1}{4} - \frac{1}{2}$

Abs MIN $(\frac{11\pi}{6}, -1/4)$ $(\sin\frac{11\pi}{6})^2 + \sin\frac{11\pi}{6} = \frac{1}{4} - \frac{1}{2}$



APAB - HW review ch. 3

3.3 #49 3.4 # 50, 51, 52, 54, 56
3.5 #5, 17

3.3 #49 $f(T) = T^2 \sin T \ [0, 2\pi]$

Despite the Directions... You DO not have a "computer algebra system" so you have to derive by hand... then graph on the calculator

(A) $F'(T) = T^2 \cos T + \sin T (2T)$

$F'(T) = T^2 \cos T + 2T \sin T$ = Y_1

(C) CRIT. pts

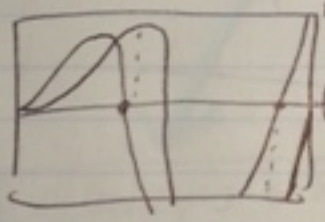
$0 = T^2 \cos T + 2T \sin T$

PUT IN CALC ...
$Y_1 = X^2 \cos X + 2X \sin X$
2ND CALC ZERO

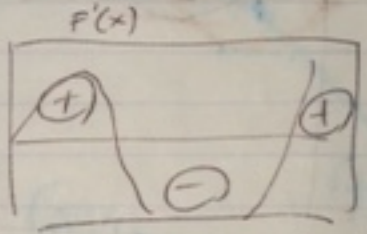
$T = 0, 2.288, 5.086$
(8) (7)

(B) $Y_1 = X^2 \sin X$

$Y_2 = X^2 \cos X + 2X \sin X$



(D)



F' (+) $(0, 2.288) \cup (5.086, 2\pi)$

F' (−) $(2.288, 5.086)$

(E) $f'(x) = 0$ when $f(x)$ has extrema

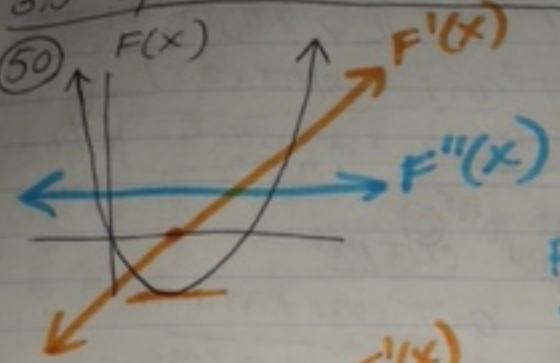
$f'(x)$ has level place where $f(x)$ has POI

APAB - HW review ch: 3

3.4 #50, 51, 52, 54, 56

3.5 #5, 17

(50)

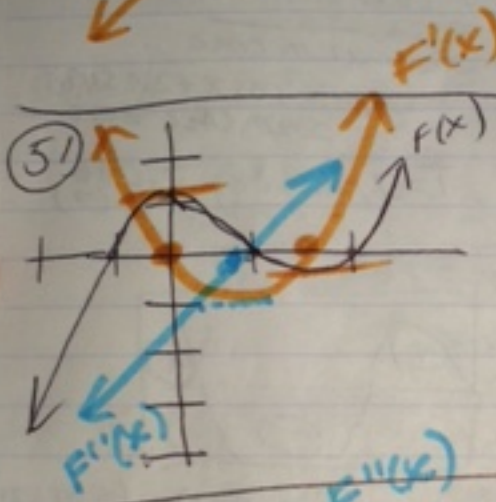


why?

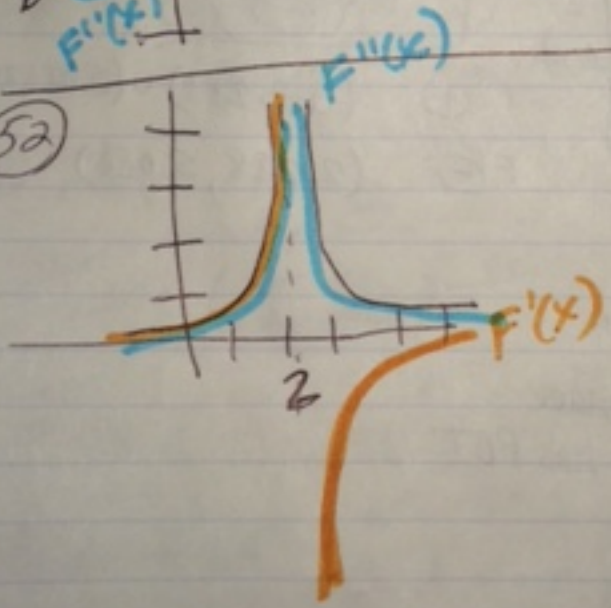
F'(x) is slope of F(x) graph

F''(x) is the slope of f'(x) graph

(51)



(52)



APAB - ch. 3 HW review

3.4 #54, 56 , 3.5 #5, 17

(54) $f(0) = f(2) = 0$ ← graph has x-ints $x = 0, 2$

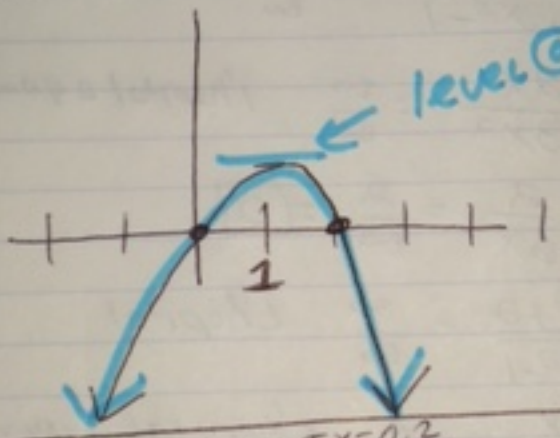
$f'(x) > 0$ if $x < 1$ (f is ~~increasing~~ to the left of $x = 1$)

$f'(1) = 0$ ← f is level at $x = 1$

$f'(x) < 0$ if $x > 1$ (f is ~~decreasing~~ to the right of $x = 1$)

$f''(x) < 0$ ← f is always concave down

(NOTE: there are MANY possible answers that meet all these requirements)



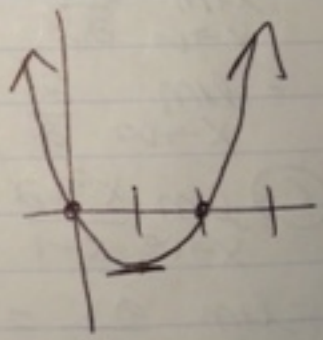
(56) $f(0) = f(2) = 0$ ← x-int $x = 0, 2$

$f'(x) < 0$ for $x < 1$ ← decreasing left of $x = 1$

$f'(1) = 0$ ← flat @ $x = 1$

$f'(x) > 0$ for $x > 1$ ← increasing right of $x = 1$

$f''(x) > 0$ ← concave up always



(AGAIN - there are MANY right answers)

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3.5 #5, 17

(5) $f(x) = \frac{x}{x^2+2}$ ← HA: $y = 0$ b/c TOP Deg < bottom Deg

SO IT'S B or D

$f(1) = \frac{1}{1+2} = \frac{1}{3}$ ...

which means it's NOT B (which has a point around (1, 1.75))

(D)

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(17) (A) $\lim_{x\to\infty} \frac{x^2+2}{x^3-1} = \frac{\infty}{\infty}$ use l'hopital

$= \lim_{x\to\infty} \frac{2x}{3x^2} = \frac{\infty}{\infty}$ l'hopital again

$= \lim_{x\to\infty} \frac{2}{6x} = \frac{2}{\infty} = \boxed{0}$

(B) $\lim_{x\to\infty} \frac{x^2+2}{x^2-1} = \frac{\infty}{\infty}$ l'hopital

$= \lim_{x\to\infty} \frac{2x}{2x} = \frac{\infty}{\infty}$ l'hopital again

$= \lim_{x\to\infty} \frac{2}{2} = \boxed{1}$

(C) $\lim_{x\to\infty} \frac{x^2+2}{x+1} = \frac{\infty}{\infty}$ l'hopital

$= \lim_{x\to\infty} \frac{2x}{1} = \frac{\infty}{1} = \boxed{\infty}$

(You CAN Also just use the Algebra of horizontal asymptotes to figure this out)