

## Chapter 3 Review

Directions: The exam will consist of 9 multiple-choice questions and 2 free response questions on a variety of topics. All questions will be from released AP examinations. THIS REVIEW also has 18 multiple-choice questions (from a test generator) and 4 free-response questions (from released AP exams). BOTH SECTIONS WILL BE NON-CALCULATOR. (Although you will need a calculator for the review multiple-choice.)

1	<p>Find the slope of the tangent line to</p> $h(x) = \sqrt{x+3} \text{ at the point } (6, 3).$ <p>A. <math>\frac{1}{6}</math>            B. 6            C. 3            D. <math>\frac{1}{3}</math></p> <p>E. None of the above</p>
2	<p>What is the derivative of <math>y(x) = -2 \sin x</math>?</p> <p>A. <math>-2 \sin x</math>            B. <math>-2 \cos x</math>            C. <math>2 \cos x</math>            D. <math>2 \sin x</math></p> <p>E. None of the above</p>
3	<p>What is <math>\frac{d}{dx} [(-3x^2 - 4x + 3)(\sin x)]</math>?</p> <p>A. <math>-3x^2 \sin x - 10x \sin x - \sin x - 3x^2 \cos x - 4x \cos x + 3 \cos x</math></p> <p>B. <math>-6x \sin x - 4 \sin x - 3x^2 \cos x - 4x \cos x + 3 \cos x</math></p> <p>C. <math>-3x^2 \sin x - 10x \sin x - \sin x</math></p> <p>D. <math>-6x \sin x - 4 \sin x + 3x^2 \cos x - 4x \cos x + 3 \cos x</math></p> <p>E. None of the above</p>

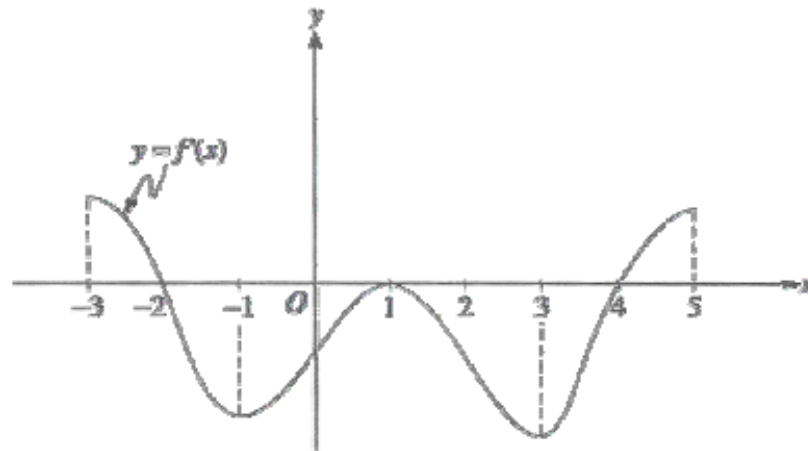
<p>4</p>	<p>If <math>y^4 = -3x^2 + 4x - 4</math>, find <math>\frac{dy}{dx}</math></p> <p>A. <math>\frac{dy}{dx} = \frac{-6x + 4}{3y}</math></p> <p>B. <math>\frac{dy}{dx} = -6x + 4</math></p> <p>C. <math>\frac{dy}{dx} = \frac{-6x + 4}{y^4}</math></p> <p>D. <math>\frac{dy}{dx} = \frac{-6x + 4}{4y^3}</math></p> <p>E. None of the above</p>
<p>5</p>	<p>If <math>2x^2y + 2y^3 = 4</math>, find <math>\frac{dy}{dx}</math></p> <p>A. <math>\frac{dy}{dx} = \frac{2x^2 + 6y^2}{-4xy}</math></p> <p>B. <math>\frac{dy}{dx} = 2x^2 + 6y^2</math></p> <p>C. <math>\frac{dy}{dx} = \frac{-4xy}{2x^2 + 6y^2}</math></p> <p>D. <math>\frac{dy}{dx} = -4xy</math></p> <p>E. None of the above</p>
<p>6</p>	<p>Find the equation of the tangent line to <math>y^4 = 3x^2 - 3x - 5</math> going through <math>(-1, -1)</math>.</p> <p>A. <math>y = \frac{-9}{4}x + \frac{5}{4}</math></p> <p>B. <math>y = \frac{-9}{4}x - \frac{5}{4}</math></p> <p>C. <math>y = \frac{9}{4}x - \frac{5}{4}</math></p> <p>D. <math>y = \frac{9}{4}x + \frac{5}{4}</math></p> <p>E. None of the above</p>

7	<p>One train travels east at 150 mph towards Challenge City, while a second train travels south at 160 mph away from Challenge City. At time <math>t = 0</math>, the first train is 10 miles west and the second train is 80 miles south. Find the rate at which the distance between the trains is changing at time <math>t = 30</math> minutes.</p> <p>A. 91.778 mph B. 18.529 mph C. 240.013 mph D. 78.75 mph E. None of the above</p>
8	<p>A 16 ft ladder leans against a wall. The bottom of the ladder is 2 feet from the wall at time <math>t = 0</math> and slides away from the wall at a rate of 1 ft/s. Find the velocity of the ladder at time <math>t = 3</math> seconds.</p> <p>A. 3.04 ft/s B. -3.04 ft/s C. 0.329 ft/s D. -0.329 ft/s E. None of the above</p>
9	<p>Find all critical points of <math>y(x) =  5x + 2 </math>.</p> <p>A. <math>x = -2/5</math> B. <math>x = 3/5</math> C. There are no critical points. D. <math>x = -7/5</math> E. None of the above</p>
10	<p>Find all critical points of <math>b(x) = \frac{x^2}{4x - 2}</math></p> <p>A. <math>x = 1/2</math> B. <math>x = 0</math> C. <math>x = 1/2</math> and <math>x = 1</math> D. <math>x = 1</math> and <math>x = 0</math> E. None of the above</p>

11	<p>Find all critical points of <math>k(x) = -2x^3 - 13x^2 - 8x + 19</math>.</p> <p>A. <math>x = -1</math> and <math>x = -4</math>.</p> <p>B. <math>x = -1/3</math> and <math>x = -4</math>.</p> <p>C. <math>x = -1/3</math> and <math>x = -4/3</math>.</p> <p>D. <math>x = -1</math> and <math>x = -4/3</math>.</p> <p>E. None of the above</p>
12	<p>Given <math>u(x) = \frac{e^x}{x - 2}</math>.</p> <p>What is <math>u'(x)</math>?</p> <p>A. <math>\frac{xe^x - 3e^x}{x^2 - 4x + 4}</math></p> <p>B. <math>\frac{xe^x - 2e^x + xe^x - 3e^x}{xe^x - 2e^x}</math></p> <p>C. <math>\frac{xe^x - 2e^x + xe^x - 3e^x}{x^2 - 4x + 4}</math></p> <p>D. <math>\frac{xe^x - 3e^x}{xe^x - 2e^x}</math></p> <p>E. None of the above</p>
13	<p>Find the intervals on which <math>t(x) = x^3 + 9x^2 + 24x - 2</math> is increasing or decreasing.</p> <p>A. Increasing: <math>(-\infty, -4) \cup (-2, \infty)</math> Decreasing: <math>(-4, -2)</math></p> <p>B. Increasing: <math>[-4, -2]</math> Decreasing: <math>(-\infty, -4) \cup [-2, \infty)</math></p> <p>C. Increasing: <math>(-\infty, -4) \cup [-2, \infty)</math> Decreasing: <math>[-4, -2]</math></p> <p>D. Increasing: <math>(-4, -2)</math> Decreasing: <math>(-\infty, -4) \cup (-2, \infty)</math></p> <p>E. None of the above</p>

14	<p>Suppose <math>h(-1) = 0</math> and <math>(hp)'(-1) = 44</math>. Find <math>p(-1)</math> assuming <math>h'(-1) = 11</math>.</p> <p>A. <math>p(-1)=55</math> B. <math>p(-1)=33</math> C. <math>p(-1)=4</math> D. <math>p(-1)=484</math></p> <p>E. None of the above</p>
15	<p>Apply the first derivative test on <math>q(x) = x^3 - 3x^2 - 24x - 4</math> to find all local maximum and minimums.</p> <p>A. <u>No Local Maximum or Minimum.</u> B. <u>Local Maximum: <math>x = -2</math> and <math>x = 4</math></u> C. <u>Local Maximum: <math>x = -2</math>, Local Minimum <math>x = 4</math></u> D. <u>Local Maximum: <math>x = 4</math>, Local Minimum <math>x = -2</math></u></p> <p>E. None of the above</p>
16	<p>Find the intervals on which <math>b(x) = -x^3 + 12x + 4</math> is concave up or down.</p> <p>A. <u>Up: <math>(-\infty, 2] \cup [-2, \infty)</math> and Down: <math>[2, -2]</math></u> B. <u>Up: <math>(-\infty, 0)</math> and Down: <math>(0, \infty)</math></u> C. <u>Up: <math>(-\infty, 2) \cup (-2, \infty)</math> and Down: <math>(2, -2)</math></u> D. <u>Up: <math>(0, \infty)</math> and Down: <math>(-\infty, 0)</math></u></p> <p>E. None of the above</p>
17	<p>Find all inflection points of <math>d(x) = 3x^5 - 15x^4 + 30x^3 - 30x^2 + 4x - 5</math>.</p> <p>A. <math>x = 1</math> B. There are no inflection points. C. <math>x = -1</math> D. <math>x = 1, x = -1</math></p> <p>E. None of the above</p>
18	<p>Apply the second derivative test on <math>a(x) = -x^3 - 6x^2 - 12x + 3</math> to find all local maximum and minimums.</p> <p>A. Local Maximum: <math>x = 2</math> B. Local Minimum: <math>x = 2</math> C. Local Maximum: <math>x = 3</math> D. The test is inconclusive.</p> <p>E. None of the above</p>

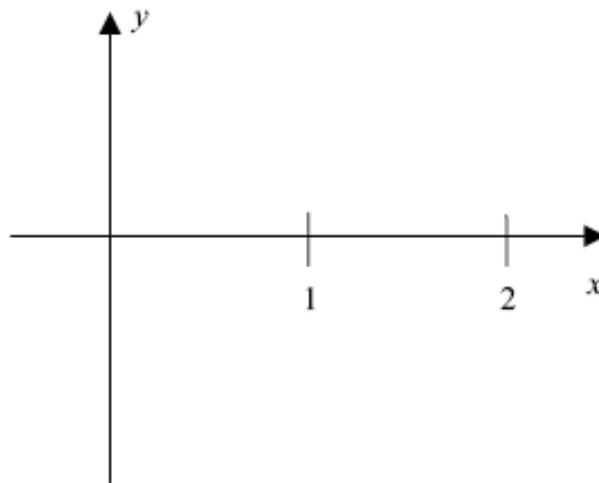
1 1996 AB1



Note: This is the graph of the derivative of  $f$ , not the graph of  $f$ .

The figure above shows the graph of  $f'$ , the derivative of a function  $f$ . The domain of  $f$  is the set of all real numbers  $x$  such that  $-3 < x < 5$ .

- For what values of  $x$  does  $f$  have a relative maximum? Why?
- For what values of  $x$  does  $f$  have a relative minimum? Why?
- On what intervals is the graph of  $f$  concave upward? Use  $f'$  to justify your answer.
- Suppose that  $f(1) = 0$ . In the  $xy$ -plane provided, draw a sketch that shows the general shape of the graph of the function  $f$  on the open interval  $0 < x < 2$ .



2

**1971 AB3**

Consider  $f(x) = \cos^2 x + 2 \cos x$  over one complete period beginning with  $x = 0$ .

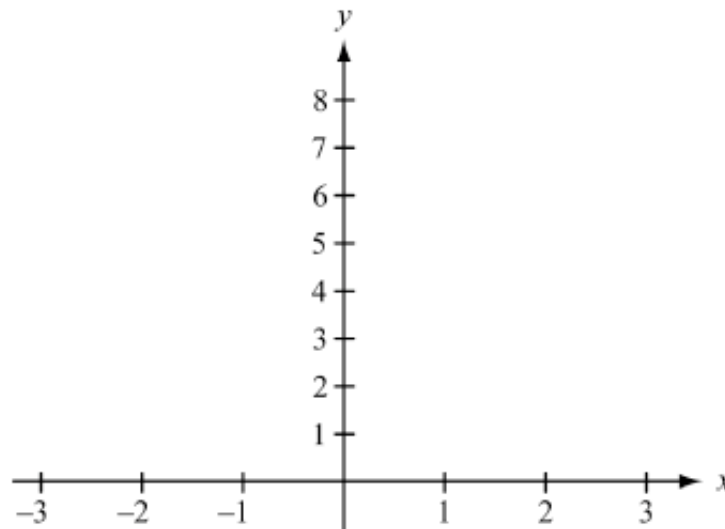
- (a) Find all values of  $x$  in this period at which  $f(x) = 0$ .
- (b) Find all values of  $x$  in this period at which the function has a minimum. Justify your answer.
- (c) Over what intervals in this period is the curve concave up?

3 **1984 AB4/BC3**

A function  $f$  is continuous on the closed interval  $[-3, 3]$  such that  $f(-3) = 4$  and  $f(3) = 1$ . The functions  $f'$  and  $f''$  have the properties given in the table below.

$x$	$-3 < x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x < 3$
$f'(x)$	Positive	Fails to exist	Negative	0	Negative
$f''(x)$	Positive	Fails to exist	Positive	0	Negative

- What are the  $x$ -coordinates of all absolute maximum and absolute minimum points of  $f$  on the interval  $[-3, 3]$ ? Justify your answer.
- What are the  $x$ -coordinates of all points of inflection of  $f$  on the interval  $[-3, 3]$ ? Justify your answer.
- On the axes provided, sketch a graph that satisfies the given properties of  $f$ .





4

**1994 AB 1**

Let  $f$  be the function given by  $f(x) = 3x^4 + x^3 - 21x^2$ .

- (a) Write an equation of the line tangent to the graph of  $f$  at the point  $(2, -28)$ .
  
- (b) Find the absolute minimum value of  $f$ . Show the analysis that leads to your conclusion.
  
- (c) Find the  $x$ -coordinate of each point of inflection on the graph of  $f$ . Show the analysis that leads to your conclusion.