APAB S.Hogan
Chapter Homework Assignment

Name: ______________________________

APAB Chapter 2 HW

Due on _________ (day AFTER Chapter Exam) at the start of the period.

Directions: Use this sheet as a cover sheet for your homework assignment. If you do not staple this cover sheet to the front of your assignment, you will receive a zero. You may submit your HW early but NO LATE HW will be accepted.

Assignment:

Section 2.1: pp.103-104#1-41EOO (11Q)
Section 2.2: pp.115-117#5-65EOO (16Q)
Section 2.3: pp.126-127#1-53EOO, 59-67odd, 85-87 (22Q)
Section 2.4: pp.137-138#3-31EOO, 41-77EOO, 87, 89 (20Q)
Section 2.5: pp.146-147#1-41EOO, 45-61EOO (16Q)
RR Packet: Worksheets 3.2, 3.3, 3.4, 3.5, 3.8 (all problems)

NOTE: "EOO" means "Every Other Odd" (so do 15, not 17, 19, not 21…etc)

Grading:

Item	Possible Points	Score
All Problems Completed	2	
Problems Labeled and in Numerical Order, Page Numbers are Labeled	2	
First Random Problem (correct with sufficient work)	4	
Second Random Problem (correct with sufficient work)	4	
Third Random Problem (correct with sufficient work)	4	
Fourth Random Problem (correct with sufficient work)	4	
TOTAL	20	

Basics of Solving Related Rates Problems S.Hogan
Using the "DREDS" Method

	What the letter stands for…	**What you need to do…**
D	Diagram	Draw a diagram based on the given information. Sometimes, this is provided, which rocks. Label the values on your diagram with variables. DO NOT put numbers on your diagram unless they are CONSTANT THROUGHOUT THE PROBLEM. (For example, when a 25-ft ladder is sliding down a wall, the length of the ladder is a constant 25 feet.)
R	Rates	Label your diagram with the rates that you know. Note that these rates are generally d(variable you assigned in your diagram)/dt. Also note that rates are positive when that variable is increasing in value and negative when that variable is decreasing in value.
E	Equation	Write an equation relating the variables in your problem. Often, you want this equation to be in two variables even though the basic equation that you need is in more than two variables. This means that you need to find some other equation to relate some of your variables and then use substitution to replace things you don't want anymore.
D	Derive	Take the derivative of the pretty equation that you just made. Usually, you will be taking the derivative of your equation with respect to time.
S	Substitute Specific Stuff (yeah alliteration!)	Plug in the values that you know and solve for the value you that you've been asked to find. Sometimes, you'll be missing other values that you need. If this happens, look back at the equations that were in your "E" step and find a way to get those values.

HINTS/TRICKS OF THE TRADE:

CONE PROBLEMS: The trick to cone problems is the fact that the cross-section of the entire cone is a triangle that is similar to the cross-section of the random "full" part at the bottom of the cone. Basically, in EVERY CONE PROBLEM the r and the h are directly proportional. Use this proportion to write an equation that relates r and h. This will allow you to replace either r or h in that pesky volume equation.

TROUGH PROBLEMS:

A. Troughs with triangular bases are the same trick as cones. The width across the top of the trough will be directly proportional to the height of the trough using similar triangles. Follow the same basic steps as the cone hint.
B. Troughs with trapezoidal bases are also relatable using similar triangles. The reason that this is trickier is because the trapezoids aren't actually similar. Split the trapezoid into a rectangle in the middle and two triangles on the sides. You have to work around the rectangle in the middle of the trapezoid and pay attention to the triangles on the sides. Use those triangles to set up another similar triangle proportion.

RIGHT TRIANGLE PROBLEMS: These only come in three varieties…

A. Problems about the sides, which will require you to use Pythagorean theorem.
B. Problems about the angles, which will require you to use SohCahToa to write an equation involving trigonometric functions.
C. Problems about area, which will require you to use the formula A = (1/2)bh.

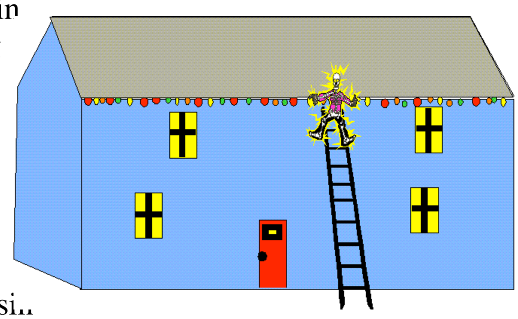
NON-RIGHT TRIANGLE PROBLEMS: These are rarer by far and will require you to use either the Law of Sines or the Law of Cosines. Usually, it requires the latter because it's harder and math teachers are mean.

PROBLEMS INVOLVING COMPASS DIRECTIONS: The directions on a compass, starting at the top and going clockwise are north, east, south, west…I remember is by saying "never eat slimy worms." It's generally helpful to draw diagrams with the objects moving in the correct direction, though it's not absolutely necessary. Remember that it's not the direction of travel that determines if a rate is positive or negative, it's whether the distance to a designated point is getting longer or shorter.

Name: ____ Worksheet 3.2

AP Calculus Related Rates – Ladder

1. A 20-meter ladder rests vertically against the side of a barn. A pig that has been hitched to the ladder starts to pull the base of the ladder away from the wall at a constant rate of 40 cm per second. Find the rate of change of the height of the top of the ladder after 30 seconds.
2. At a given instant the legs of a right triangle are 5 cm and 12 cm long. If the short leg is increasing at the rate of 1 cm/sec and the long leg is decreasing at the rate of 2 cm/sec, how fast is the area changing?
3. At a given instant the legs of a right triangle are 5 cm and 12 cm long. If the short leg is increasing at the rate of 1 cm/sec and the long leg is decreasing at the rate of 2 cm/sec, how fast is the hypotenuse changing?
4. A 13-foot ladder is leaning against a wall. If the top of the ladder slips down the wall at a rate of 2 ft/sec, how fast will the foot be moving away from the wall when the top is 5 feet above the ground. (5/6 feet/sec)
5. A ladder 25 feet long is leaning against a wall. If the lower end of the ladder is pulled away from the wall at a rate of 1 foot per second, what rate is the other end slipping down the wall when it is 24 feet from the ground?
6. A 10-foot long ladder rests against a vertical wall. If the bottom of the ladder slides away from the wall at a speed of 2 ft/se, how fast is the angle between the top of the ladder and the wall changing when the angle is 45 degrees?
7. A ladder is 26 feet long and rests on horizontal ground against a vertical wall. The foot of the ladder is pulled away from the base of the wall at the rate of 3 ft/sec. How fast is the top of the ladder sliding down the wall when the foot of the ladder is 10 feet from the wall?
8. A ladder 15 feet long is leaning against a building so that the end X is on ground level and end Y is on the wall. X is moving away from the building at a rate of .5 ft/sec. Let O be the point where the building meets the ground and assume OX and OY form a right angle.
   a. Find the rate in feet per second at which the length OY is changing when X is 9 feet from the building.
   b. Find the rate of change of the area of the triangle XOY at the same instant.
9. It is Christmas time at the Griswold house. Clark again feels compelled to put lights to make his house the shining glory of the neighborhood. He is putting up Christmas lights on the gutter on the house when Snots the dog gets his leash tangled with the bottom of the ladder. The 15-foot ladder begins to slide directly away from the house at a rate of 18 feet per second. Cousin Eddie who is six feet tall is standing beside the house directly under the ladder. At what rate will the top of the ladder hit Cousin Eddie in the head?



10. A ladder 15 feet long is leaning against a building. The bottom of the ladder is moving away from the building at the constant rate of 1/2 ft/sec.
    a. Find the rate at which the ladder is moving down the building when the bottom of the ladder is 9 feet from the building.
    b. Find the rate of change of the area of the triangle formed by the ladder, building, and ground when the bottom of the ladder is 9 feet from the building.
11. A 13-foot ladder is leaning against a house when its base starts to slide away. By the time the base is 12 feet from the house, the base is moving at a rate of 5 feet per second.
    a. How fast is the ladder sliding down the wall at that instant?
    b. How fast is the area of the triangle formed by the ladder, wall, and ground changing?

Name: ______________________________________ Worksheet 3.3
AP Calculus Related Rates – Shadows & Pulleys

1. Sand pouring from a chute forms a conical pile whose height is equal to the diameter. If the height increases at a constant rate of 5 ft/min, at what rate is the sand pouring from the chute when the pile is 10 ft high?
2. A boat is pulled into a dock by means of a rope attached to a pulley on the dock. The rope is attached to the bow of the boat at a point 10 feet below the pulley. How fast must the rope be pulled if we want the boat to approach the dock at a rate of 12 ft/min at the instant when 125 ft of rope is out?
3. A boy 4.5 feet tall walks toward a light 10 feet above the ground at the rate of 6 feet per second. How fast is his shadow changing in length?
4. A boy 4.5 feet tall walks toward a light 10 feet above the ground at the rate of 6 feet per second. How fast is the shadow of his head moving?
5. A man on a pier pulls in a rope attached to a small boat at the rate of 1 foot/sec. If his hands are 10 feet above the place where the rope is attached, how fast is the boat approaching the pier when there is 20 feet of rope out?
6. A boat is being pulled to shore by a rope attached to a windlass on top of a pier. The height of the windlass above the water is 6 m and the rope is being wound at a rate of 4 meters per minute. How fast is the boat approaching the shore when it is 12 m away?
7. A light is being raised up a pole. The light shines on a man, 2 meters tall and 15 meter from the pole, casting a shadow on the ground. At a certain moment the light is 20 m off the ground, rising at 3 meters per min. How fast is the shadow shrinking at that moment?
8. A lamppost is 20 meters high. A man of height 2 meter passes under the lamppost and continues walking at 3 meters per second. At what rate is the length of the shadow increasing? How fat is the tip of the shadow moving?
9. One end of a rope is tied to a crate. The other end of the rope is passed over a pulley 6 meter above the floor and tied 1 meter above the floor to the back of a forklift. If the rope is taut and the forklift moves at the rate of 1.5 m/sec, how fast is the box rising when the forklift is 4 m from the vertical formed by the rope and pulley?
10. A boat is pulled in to a dock by a rope with one end attached to the bow of the boat, the other end passing through a ring attached to the dock at a point 4 feet higher than the bow of the boat. If the rope is pulled in at the rate of 2 ft/sec, how fast is the boat approaching the dock when 10 feet of rope are out?
11. A man 6 feet tall walks at the rate of 5 ft/sec toward a streetlight that is 16 feet above the ground. At what rate is the tip of his shadow moving? At what rate is the length of his shadow changing when he is 10 feet from the base of the light?
12. A dinghy is pulled toward a dock by a rope from the bow through a ring on the dock 6 feet above the bow. If the rope is hauled in at the rate of 2 feet per second, how fast is the boat approaching the dock when 10 feet of rope are out?

Name: ____ Worksheet 3.4
AP Calculus Related Rates – Numbers and Spheres

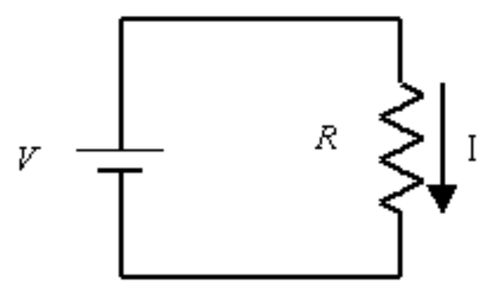
1. If $x^2+3xy+y^2=1$ and $\frac{dy}{dt}=2$, find $\frac{dx}{dt}$ when $y=1$.
2. Find $\frac{dy}{dt}$ if $y^2=x^2(x+1)$, $\frac{dx}{dt}=a$, $x=1$, and $y=-\sqrt{2}$.
3. Find $\frac{dy}{dt}$ if $y=\frac{1}{2}(e^x+e^{-x})$, $\frac{dx}{dt}=1$, and $x=1$.
4. A point moves along the upper half of the curve $y^2 = 2x + 1$ in such a way that $\frac{dx}{dt}=\sqrt{2x+1}$. Find $\frac{dy}{dt}$ when $x=4$.
5.
6. A spherical balloon is inflated with helium at the rate of 100π ft^3 / min. How fast is the balloon's radius increasing at the instant the radius is 5 ft? How fast is the surface area increasing?
7. Suppose as above a balloon is being inflated with gas at a rate of 3 cc/sec. At what rate is the area increasing when the radius is 14 cm?
8. Air is being pumped into a spherical balloon so that its volume increases at a rate of 80 cm^3/sec. How fast is the radius increasing when the diameter is 60 cm?
9. A spherical balloon is inflated with gas at the rate of 100 cubic feet per minute. Assuming that the gas pressure remains constant, how fast is the radius of the balloon increasing at the instant when the radius is 3 feet? How fast is the surface area increasing at this instant?
10. A spherical balloon is leaking air at a rate of 2 cm 3/ sec. How fast is the radius decreasing at the instant when the radius is 5 cm?
11. A spherical snowball is melting in such a way that its volume is decreasing at a rate of 1cm^3/min. At what rate is the diameter decreasing when the diameter is 10 cm?
12. A spherical balloon is to be deflated so that its radius decreases at a constant rate of 15 cm/min. At what rate must air be removed when the radius is 9 cm?
13. Let $u = (x-1)^3$ and $v = u = (x+1)^3$ where x is a differentiable function of 'f'. If $\frac{du}{dt}=6$ when $\frac{dx}{dt}=\frac{1}{2}$, what is $\frac{dv}{dt}$ then?
14. Multiple part problem:
    a. Describe the relationship between the rate of change of the volume of a sphere with respect to time and the surface area of the sphere.
    b. If a spherical balloon is being inflated with helium at a rate of 7 cubic feet per minute, determine the rate of change of the surface area of the balloon when the radius of the balloon is 10 feet.
    c. If a spherical balloon is being inflated at a constant rate, determine the rate of change of the surface area with respect to time as a function of the balloon's radius.
    d. When the magnitude of the rate of change of the surface area of the balloon with respect to time in part c is one-third of the magnitude of the rate of change of the volume of the balloon with respect to time, determine the radius of the balloon.

15. Lisa was a very naughty girl this year. Santa Claus was very disappointed in her, so he filled her cylindrical Christmas stocking with his magic powdered coal. Her cylindrical stocking is 12 inches high and 6 inches in diameter. The stocking fills at a rate of 5 cubic inches per minute.
    a. What is the volume of Lisa's stocking?
    b. What is the volume of the coal when it is 8 inches high?
    c. At what rate is the height of the coal in the stocking changing when the height is 8 inches?

Name: ____ Worksheet 3.5

AP Calculus Related Rates – Vehicle

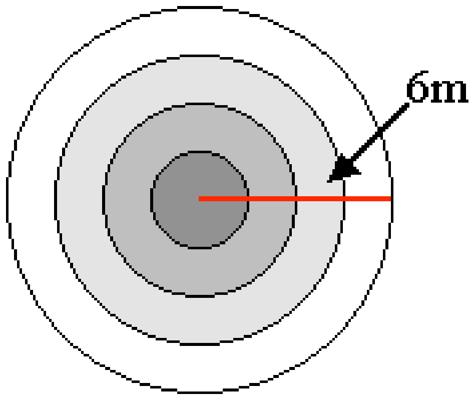
1. Two cars start moving from the same point. Car A is traveling east at 100 km/hr and car B is traveling south at 80 km/hr. At what rate is the distance between them increasing 4 hours later?
2. Car A is traveling north at 80 km/hr and car B is traveling west at 110 km/hr. Both are headed for the intersection of the two roads. At what rate are the cars approaching each other when car A is .4 km and car B is .7 km from the intersection?
3. A plane flying horizontally at an altitude of 2 km and a speed of 700 km/hr passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 6 km from the station.
4. A ship is 400 miles directly south of Tahiti and is sailing south at 20 miles/hour. Another ship is 300 miles east of Tahiti and is sailing west at 15 miles/hour. At what rate is the distance between the ships changing?
5. A plane flying at an altitude of 1 mile is 2 miles distant from an observer, measured along the ground, and flying directly away from the observer at 400 mph. How fast is the angle of elevation changing?
6. At noon of a certain day, ship A is 60 miles due north of ship B. If ship A sails east at 15 miles per hour and B sails north at 12 miles per hour, determine how rapidly the distance between them is changing 4 hours later. Is it decreasing or increasing?
7. One ship leaves port and steams due north at 10 knots. Three hours later another ship leaves the same port and steams due west at 30 knots. How fast is the distance between them increasing when the first ship has been out of port for 5 hours? (A knot is a nautical mile per hour.)
8. Two cars start from the same point. One travels south at 60mi/hr and the other travels west at 25mi/hr. At what rate is the distance between them increasing two hours later?
9. At noon, ship A is 150 km west of ship B. Ship A is sailing east at 35km/hr and ship B is sailing north at 25 km/hr. How fast is the distance changing between them at 4:00 pm?
10. A man starts walking north at 4 ft/s from point A. Five minutes later a woman starts walking south at 5 ft/s from a point 500 ft due east of A. At what rate are they separating 15 min after the woman starts?
11. An airplane, flying east at 400 mph, goes over a certain town at 11:30 am and a second plane, flying northeast at 500 mph, goes over the same town at noon. How fast are they separating at 1:00 pm?
12. At noon ship **A** was 12 nautical miles due north of ship **B**. Ship A was sailing south at 12 knots (nautical miles per hour) and continued to do so all day. Ship B was sailing east at 8 knots and continued to do so all day. How rapidly was the distance between the ships changing at (a) noon, and (b) one hour later? The visibility that day was 5 nautical miles. Did the ships ever catch sight of each other?
13. A rocket, rising vertically, is tracked by a radar station that is on the ground .5 miles from the launch pad. At what rate is the elevation angle changing when the rocket is 3000 feet up and rising vertically at 500 ft/sec?
14. An aircraft is climbing at a 30-degree angle to the horizontal. How fast is the aircraft gaining altitude if its speed is 500 mi/hr?
15. Two commercial jets at 40,000 ft are flying at 520 mi/hr along straight-line courses that cross at right angles. How fast is the distance between the airplanes closing when airplane A is 5 mi from the intersection point and airplane B is 12 mi from the intersection point? How fast is the distance closing at any time?
16. Ohm's Law for electrical circuits like the one shown states that V = I R where V is the voltage, I is the current in amperes, and R is the resistance in ohms. Suppose that V is increasing at the rate of 1 volt/second while I is decreasing at the rate of 1/3 amp/sec. Let t denote time in seconds.
    a. What is the value of dV/dt?
    b. What is the value of dI/dt?
    c. What equation related dR/dt and dI/dt?
    d. Find the rate at which R is changing when V=12 volts and I = 2 amps. Is R increasing or decreasing?



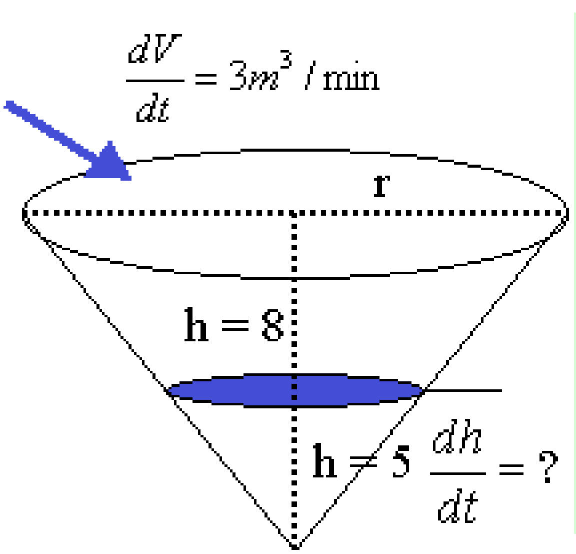
Name: ____ Worksheet 3.8

AP Calculus Related Rates – Circle, Cone, Trough

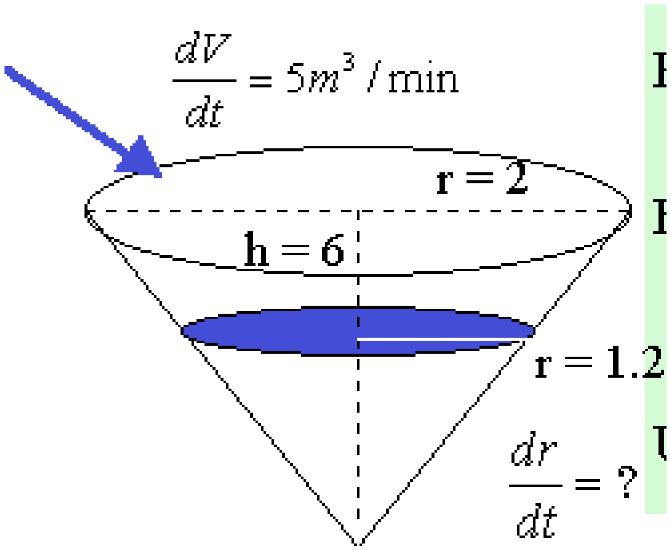
1. A golf ball chipped into a water hazard creates a circular ripple effect. If the radius of the ripple is increasing at 0.8 m/s, how fast is the area changing when the radius is 6m? ($9.6\pi\ m^2$ / sec.)



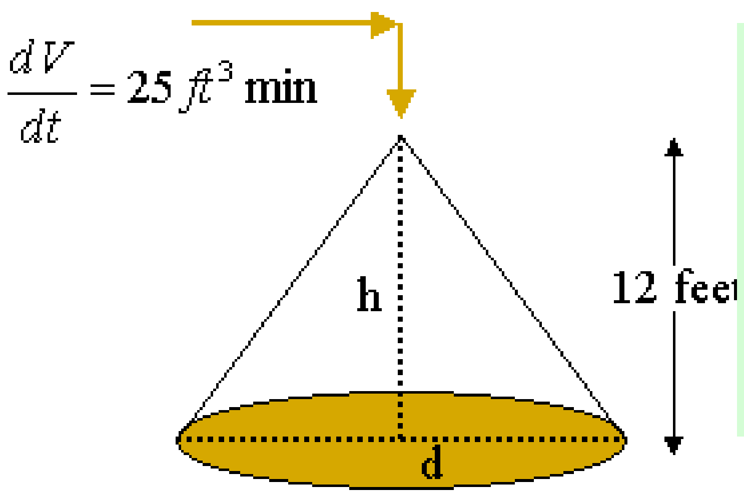
2. A tank in the shape of a right circular cone with altitude 8m and a base radius of 3m is being filled with water at a rate of 3000 L/min ($3\ m^3$ / min). How fast is the surface of the water rising when the depth is 5m? ($64/75\pi$ m/min)



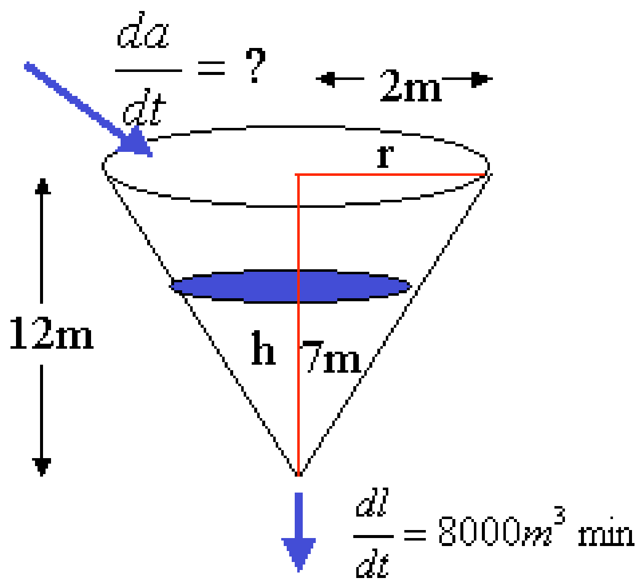
3. A water tank has the shape of an inverted cone with a base radius of 2m and a height of 6m. If water is being pumped into the tank at a rate of 5 m^3 / min, find the rate at which the radius of the surface of the water is growing when the radius is 1.2 m. ($5/1.44\pi$ m/min)



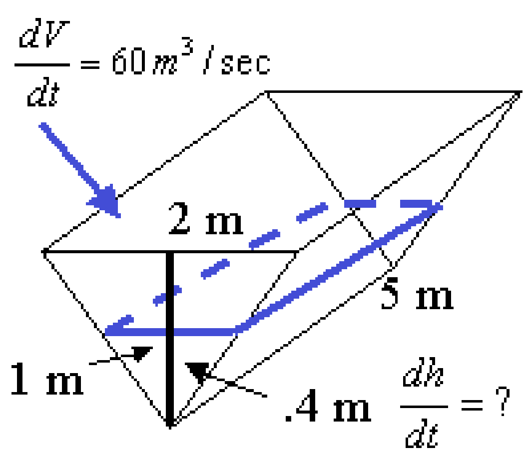
4. Gravel is being dumped from a conveyor belt at a rate of 25 ft^3 per minute and it forms a pile in the shape of a cone whose base diameter is twice the height. How fast is the height increasing when the pile is 12 feet high? ($25/144\pi$ ft/min)



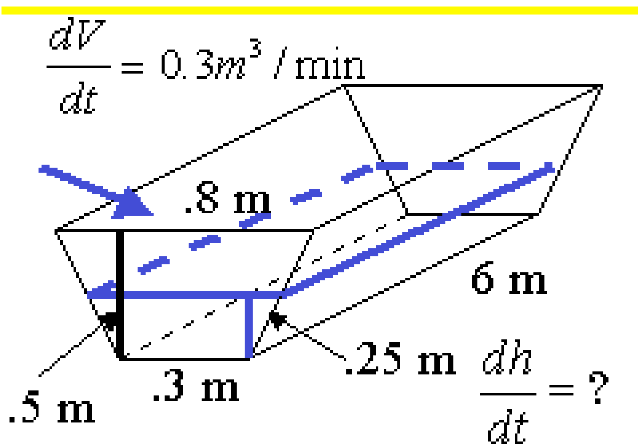
5. Water is leaking out of an inverted conical tank at a rate of 8,000 cubic meters per minute at the same time water is being pumped into the tank at a constant rate. The tank has a height of 12m and the diameter of the top is 4m. If the water is rising at the rate of 40 cm/min when the height of the water is 7m, find the rate at which water is being pumped into the tank. ($8000 + 19.6\pi/9 \ m^3$ / min)



6. The cross section of a water trough is an inverted isosceles triangle with its top edge horizontal. The triangle has a base of 2m and a height of 1 m. The trough is 5m long. Water is being pumped into the trough at a rate of 60 cubic meters per second. How fast is the water rising when it is 40 cm deep? (15m/sec)



7. A water trough is 6m long and its cross-section has the shape of an isosceles trapezoid that is 30 cm wide at the top and 30 cm wide at the bottom, 80 cm wide at the top, and 50 cm high. If the trough is being filled with water at a rate of 0.3 cubic meters per minute, how fast is the water rising when the water is 25 cm deep? (1/11 m/min)



8. A trough 10 ft long has a cross section that is an isosceles triangle 3 ft deep and 8 ft across. If water flows in at the rate of 2 ft^3/min, how fast is the surface rising when the water is 2 ft deep? (0.45 in/min)

9. Oil spilled from a ruptured tanker spreads in a circle whose area increases at a constant rate of 6 km^2/hr. How fast is the radius of the spill increasing when the area is 9km^2. ($1/\sqrt{\pi}$)

10. A conical water tank with vertex down has a radius of 10 ft at the top and is 24 ft high. If water flows into the tank at a rate of 20 ft^3/min, how fast is the depth of the water increasing when the water is 16 feet deep. ($9/20\pi$ ft/min)

11. Sand pouring from a chute forms a conical pile whose height is equal to the diameter. If the height increases at a constant rate of 5 ft/min, at what rate is the sand pouring from the chute when the pile is 10 ft high? (125π ft^3/min)

12. Water is flowing at a rate of 5 cubic feet per minute into a tank in the form of a cone of altitude 20 feet and base radius of 10 feet and with its vertex in the downward direction. Find the rate at which the uncovered surface of the conical tank is decreasing when the water is 8 feet deep.

13. A pile of sand being dumped forms a right circular cone in which the altitude is 2/3 the diameter. If the sand is dumped at 3 cubic feet per second, find the rate of increase of the diameter of the pile when it is 6 feet high.

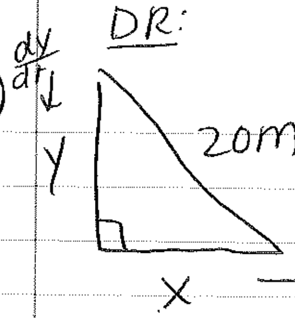
14. A trough that is 12 feet long and 2 feet high is 2 feet wide at the top and has triangular ends. If water is put into the tank at the rate of 1 cubic foot per minute, how fast is the depth increasing when it is 1.5 feet deep.

15. A circular oil slick of uniform thickness is caused by a spill of one cubic meter of oil. The thickness of the oil spill is decreasing at the rate of 0.1 cm per hour. At what rate is the radius of the slick increasing when it is 8 m?

16. Sand falls from a conveyor belt onto a conical pile at the rate of 10 cubic feet per minute. The radius of the base of the pile is always equal to twice its altitude. How fast is the altitude of the pile increasing when the pile is 8 feet high?

3.2 RR

(1) DR:



$\frac{dx}{dt} = .4\,m/s$

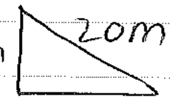
FIND $\frac{dy}{dt}$ After 30 sec.

E: $x^2+y^2=20^2$

D: $2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$

$\frac{dy}{dt} = \frac{-\cancel{2}x(dx/dt)}{\cancel{2}y} = \frac{-x(dx/dt)}{y}$

S: when $t = 30$ sec



b/c IT'S A 3-4-5 △

$\frac{dy}{dt} = \frac{-12(.4)}{16}$

$= -\frac{3}{4}(.4) = \boxed{-.3\,m/s}$

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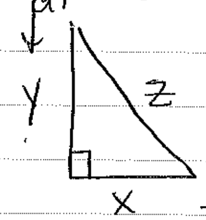
(3) call x = short leg
y = long leg

DR: $\frac{dy}{dt} = -2\,cm/s$



$\frac{dx}{dt} = 1\,cm/se$

FIND dz/dt when $x=5$ and $y=12$

E: $x^2+y^2=z^2$

D: $2x\,dx/dt + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$\frac{(2x(dx/dt)+2y(dy/dt))}{2z} = \frac{dz}{dt}$

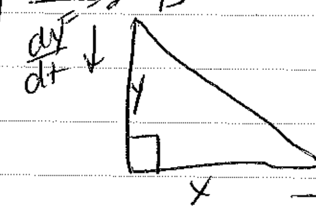
S: $x=5,\ y=12$ (so $z=13$ ... 5-12-13 △)
$dx/dt = 1 \quad dy/dt = -2$

$\frac{dz}{dt} = \frac{x(dx/dt)+y(dy/dt)}{z} = \frac{5(1)+12(-2)}{13}$

$\frac{dz}{dt} = \frac{5-24}{13} = \boxed{\frac{-19}{13}\ cm/s}$

(2) x = short leg
y = long leg

DR: $\frac{dy}{dt} = 2\,cm/s$



$\frac{dx}{dt} = 1\,cm/s$

FIND $\frac{dA}{dt}$ when $x=5$, $y=12$

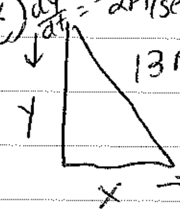
E: $A = \frac{1}{2}xy$

D: $\frac{dA}{dt} = \frac{1}{2}x\frac{dy}{dt} + y\left(\frac{1}{2}\frac{dx}{dt}\right)$

S: $\frac{dA}{dt} = \frac{1}{2}(5)(-2) + (12)(\frac{1}{2})(1)$

$= -5+6 = \boxed{1\ cm^2/sec}$

(4) $\frac{dy}{dt} = -2\,ft/sec$



$\frac{dx}{dt}$

FIND $\frac{dx}{dt}$ when $y=5$ (so $x=12$ 5-12-13 △)

E: $x^2+y^2=13^2$

D: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

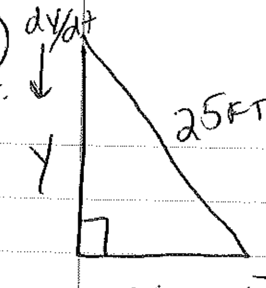
$\frac{dx}{dt} = \frac{-y(dy/dt)}{x}$

S: $\frac{dx}{dt} = \frac{-5(-2)}{12} = \frac{10}{12} = \boxed{\frac{5}{6}\ FT/sec}$

3.2RR

(2)

(5) $\frac{dy}{dt}\downarrow$ DR:



$\rightarrow \frac{dx}{dt} = 1\,FT/s$

E: $x^2+y^2=25^2$

D: $2x\,dx/dt + 2y\,dy/dt = 0$

$dy/dt = \frac{-x(dx/dt)}{y}$

S: when ~~[illegible]~~ $y=24$ (so $x=7$... 7-24-25 △)

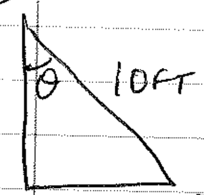
$\frac{dy}{dt} = \frac{-7(1)}{24} = \frac{-7}{24}$ FT/s

~~[illegible]~~

So sliding DOWN at $\frac{7}{24}$ FT/s

---

(6) DR



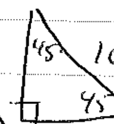
$\rightarrow dx/dt = 2\,FT/s$

Find $d\theta/dt$ when $\theta = 45^\circ$

E: $\sin\theta = \frac{x}{10}$

D: $\cos\theta \frac{d\theta}{dt} = \frac{1}{10}\,dx/dt$

S:



← oops... DON'T need IT

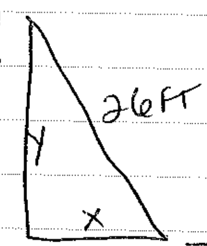
$\cos 45^\circ \frac{d\theta}{dt} = \frac{1}{10}(2)$

$\left(\frac{\sqrt{2}}{2}\right)\frac{d\theta}{dt} = \frac{1}{5}$

$d\theta/dt = \frac{1}{5}\cdot\frac{2}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{5}\ \text{Deg/sec}}$

---

(7) $\frac{dy}{dt}\downarrow$



$\rightarrow dx/dt = 3\,FT/sec$

Find $\frac{dy}{dt}$ when $x=10$ (IF $x=10$, $y=24$... IT'S a 5-12-13 △)

E: $x^2+y^2=26^2$

D: $2x\,dx/dt + 2y\frac{dy}{dt} = 0$

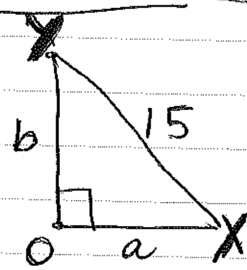
$dy/dt = \frac{-x(dx/dt)}{y}$

S: $\frac{dy}{dt} = -\frac{10(3)}{24} = \frac{-30}{24} = \frac{-5}{4}$ FT/sec

So sliding DOWN at $\frac{5}{4}$ FT/sec

---

(8)



$\rightarrow \frac{da}{dt} = .5\,FT/sec$

(A) Find db/dt when ~~a~~ $a=9$ (so $b=12$... IT'S a 3-4-5 △)

E: $a^2+b^2=15^2$

D: $2a\frac{da}{dt} + 2b\frac{db}{dt} = 0$

$db/dt = \frac{-a(da/dt)}{b}$

S: $db/dt = \frac{-9(.5)}{12} = \boxed{\frac{-9}{24}\ \text{FT/sec}}$

(B) E: $A = \frac{1}{2}ab$

D: $\frac{dA}{dt} = \frac{1}{2}a\frac{db}{dt} + b\left(\frac{1}{2}\frac{da}{dt}\right)$

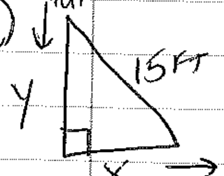
S: $\frac{dA}{dt} = \frac{1}{2}(9)\left(\frac{-9}{24}\right) + 12\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{-81}{48} + 3 = \frac{-27}{16} + \frac{48}{16}$

$\boxed{\frac{dA}{dt} = \frac{21}{16}\ \text{FT}^2/\text{sec}}$

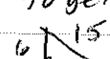
---

(9) $dy/dt \downarrow$



$\rightarrow \frac{dx}{dt} = 18\,FT/sec$

Find $\frac{dy}{dt}$ when $y=6$ (IF $y=6$, use pythag. thm to get x



$\sqrt{225-36}$
$\sqrt{189} = x$)

E: $x^2+y^2=15^2$

D: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

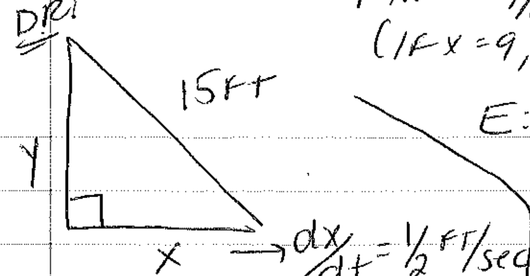
$dy/dt = \frac{-x(dx/dt)}{y}$

S: $\frac{dy}{dt} = \frac{-\sqrt{189}(18)}{6}$

$\boxed{= 3\sqrt{189}\ \text{FT/sec}}$

3.2RR

(10)



(A) FIND dy/dt when $x=9$
(IF $x=9$, $y=12$ ... 3-4-5 Δ)

E: $x^2+y^2=15^2$

D: $2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$

$dy/dt = \frac{-x(dx/dt)}{y}$

S: $\frac{dy}{dt} = \frac{-9(1/2)}{12} = \frac{-9}{24}$ FT/sec

So $\boxed{\text{Down at } 9/24 \text{ FT/sec}}$

(B) FIND dA/dt when $x=9$
(SO $y=12$... 3-4-5 Δ)

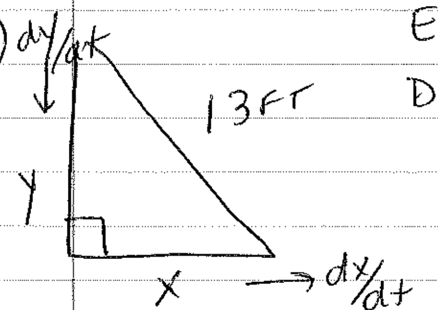
E: $A=\frac{1}{2}xy$

D: $\frac{dA}{dt} = \frac{1}{2}x\frac{dy}{dt} + y\left(\frac{1}{2}\frac{dx}{dt}\right)$

S: $\frac{dA}{dt} = \frac{1}{2}(9)\left(\frac{-9}{24}\right)+(12)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{-27}{16}+3 = \frac{-27}{16}+\frac{48}{16} = \boxed{\frac{21}{16}\text{ FT}^2/\text{sec}}$

---

(11)



E: $x^2+y^2=13^2$

D: $2x\,dx/dt+2y\,dy/dt=0$

$dy/dt = \frac{-x(dx/dt)}{y}$

S: when $x=12$, $y=5$ (IT'S 5-12-13 Δ)
and $dx/dt = 5$ FT/sec

(A) $\frac{dy}{dt} = \frac{-12(5)}{5} = \boxed{-12\text{ FT/sec}}$

$\boxed{\text{SO sliding DOWN @ 12 FT/sec}}$

(B) E: $A=\frac{1}{2}xy$

D: $\frac{dA}{dt} = \frac{1}{2}x\,dy/dt + y\left(\frac{1}{2}dx/dt\right)$

S: $\frac{dA}{dt} = \frac{1}{2}(12)(-12)+5\left(\frac{1}{2}\right)(5) = -72+\frac{25}{2} = \frac{-144}{2}+\frac{25}{2} = \frac{-119}{2}$

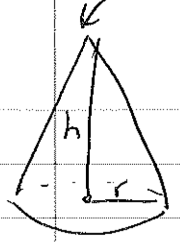
$\boxed{\frac{dA}{dt} = \frac{-119}{2}\text{ FT}^2/\text{sec}}$

$\boxed{3.3RR}$ $\frac{dV}{dt} = ??$

④

① $h = d$

$h = 2r$



$\frac{dh}{dt} = 5\ FT/min$

FIND $\frac{dV}{dt}$ when $h = 10$

E: $V = \frac{1}{3}\pi r^2 h$ $\quad h = 2r \rightarrow \frac{h}{2} = r$

replace r.

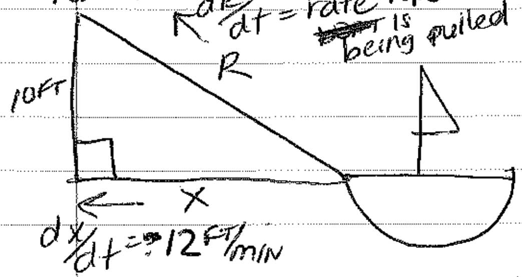
$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{\pi}{12}h^3$

D: $\frac{dV}{dt} = \frac{\pi}{4}h^2\, dh/dt$

S: $\frac{dV}{dt} = \frac{\pi}{4}(10)^2(5) = \frac{500\pi}{4} = \boxed{125\pi \frac{FT^3}{MIN}}$

---

②



$\frac{dR}{dt}$ = rate rope ~~????~~ is being pulled

$dx/dt = -12\ FT/MIN$

FIND $\frac{dR}{dt}$ when $R = 125FT$

IF $R = 125FT$, $x = \sqrt{125^2 - 10^2}$

$= \sqrt{15525} = 5\sqrt{621}$

$X = 15\sqrt{69}$

E: $x^2 + 10^2 = R^2$

D: $2x\frac{dx}{dt} + 0 = 2R\frac{dR}{dt}$

$\frac{dR}{dt} = \frac{\cancel{2}x(dx/dt)}{\cancel{2}R}$

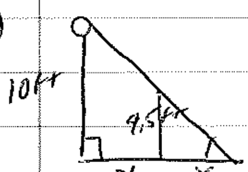
S: $\frac{dR}{dt} = \frac{15\sqrt{69}(-12)}{125} = \frac{-36\sqrt{69}}{25} \approx -11.961_{(2)}$

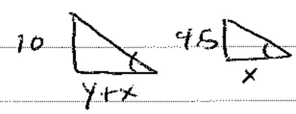
so pull the rope at

$\frac{36\sqrt{69}}{25}$ FT/min or $\approx 11.961_{(2)}$ FT/MIN

---

③





$\leftarrow dx/dt =$ how shadow changes length

$\frac{dy}{dt} = -6\ FT/sec$

E: $\frac{10}{y+x} = \frac{4.5}{x}$

$10x = 4.5y + 4.5x$

$(5.5x = 4.5y)(2)$

$11x = 9y$

D: $11\, dx/dt = 9\frac{dy}{dt}$

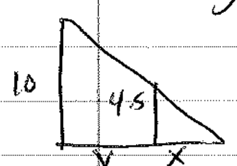
$dx/dt = \frac{9}{11}dy/dt$

S: $\frac{dx}{dt} = \frac{9}{11}(-6)$

$\boxed{\frac{dx}{dt} = -\frac{54}{11}\ FT/s}$

---

④ same diagram As #3...



length shadow tip $= s = y + x$

E: $s = y + x$

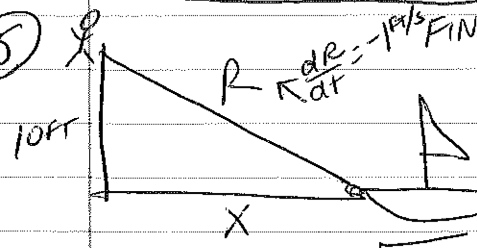
D: $\frac{ds}{dt} = \frac{dy}{dt} + \frac{dx}{dt}$

From #3

S: $\frac{ds}{dt} = -6 + \frac{-54}{11} = -\frac{66}{11} - \frac{54}{11} = \boxed{\frac{-120}{11}\ FT/s}$

---

⑤



$\frac{dR}{dt} = -1\ FT/s$ FIND $\frac{dx}{dt}$ when $R = 20$

* IF $R = 20$ then $x = \sqrt{20^2 - 10^2}$

$= \sqrt{300} = 10\sqrt{3}$

E: $x^2 + 10^2 = R^2$

D: $2x\frac{dx}{dt} = 2R\frac{dR}{dt}$

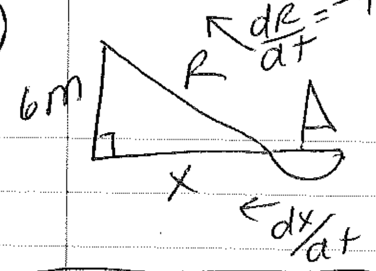
$\frac{dx}{dt} = \frac{R(dR/dt)}{x}$

S: $\frac{dx}{dt} = \frac{20(-1)}{10\sqrt{3}} = \frac{-2}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}\ FT/s$

So APPROACHING DOCK at $\frac{2\sqrt{3}}{3}$ FT/s

$\boxed{3.3RR}$

6



FIND $\frac{dx}{dt}$ when $x = 12m$ $\left(\text{IF } x=12,\ R = \sqrt{6^2+12^2} = \sqrt{36+144} = \sqrt{180} = 6\sqrt{5}\right)$

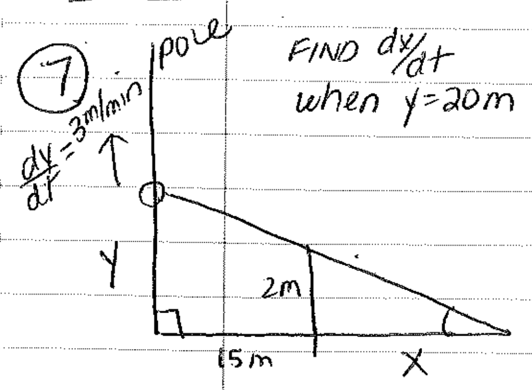
E: $x^2 + 6^2 = R^2$

D: $2x\frac{dx}{dt} = 2R\frac{dR}{dt} \rightarrow \frac{dx}{dt} = \frac{\cancel{2}R(dR/dt)}{\cancel{2}x}$

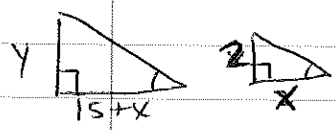
S: $\frac{dx}{dt} = \frac{6\sqrt{5}(-4)}{12} = -2\sqrt{5}$ M/MIN

$\boxed{\text{So } \underline{\text{Approaching}} \text{ Dock at } 2\sqrt{5}\ m/min}$

7



FIND dy/dt when $y = 20m$



IF $y = 20$

$\frac{20}{15+x} = \frac{2}{x}$

$20x = 30 + 2x$

$18x = 30$

$x = 30/18 = 5/3$

E: $\frac{Y}{15+X} = \frac{2}{X} \rightarrow \frac{\cancel{X}y}{\cancel{X}} = \frac{30}{X} + \frac{2X}{X} \rightarrow y = \frac{30}{x} + 2$

$y = 30x^{-1} + 2$

D: $\frac{dy}{dt} = -30x^{-2}\frac{dx}{dt}$

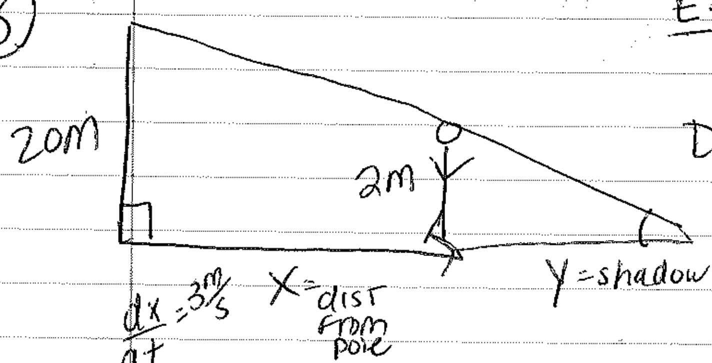
$\frac{dy}{dt} = \frac{-30}{x^2}\,dx/dt \rightarrow \frac{dx}{dt} = \frac{x^2}{-30}\frac{dy}{dt}$

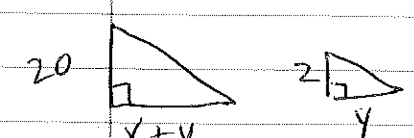
S: $\frac{dx}{dt} = \frac{(5/3)^2}{-30}\frac{dy}{dt} = \frac{25}{-270}\frac{dy}{dt}$

$\frac{dx}{dt} = \frac{-25}{270}(3) = \frac{-25}{90} = \frac{-5}{18}$ m/min

$\boxed{\text{So } \underline{\text{shrinking}} \text{ at } 5/18\ m/min}$

8





E: $\frac{20}{x+y} = \frac{2}{y} \rightarrow 20y = 2x + 2y$

$18y = 2x$

D: $18\frac{dy}{dt} = 2\,dx/dt \rightarrow \frac{dy}{dt} = \frac{2(dx/dt)}{18}$

S: $\frac{dy}{dt} = \frac{2(3)}{18} = \boxed{1/3\ m/s}$

rate length of shadow increases

tip of shadow, S, is x+y

E: $S = x + y$

D: $\frac{ds}{dt} = \frac{dx}{dt} + \frac{dy}{dt}$

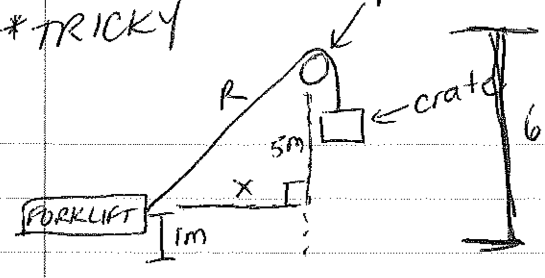
S: $\frac{ds}{dt} = 3 + 1/3 = \boxed{4/3\ m/s}$

rate tip of shadow is moving

3.3RR

(6)

(9) *TRICKY



TRICKY part is that crate is rising exactly as fast as R is being pulled (because it's a rope and every part of the rope must move at the same speed.

$\frac{dx}{dt} = 1.5\,m/s$

FIND $\frac{dR}{dt}$ when $x=4m$

IF $x=4$, then $R=\sqrt{4^2+5^2}=\sqrt{41}$

$\underline{\underline{E}}$: $x^2+5^2=R^2$

$\underline{\underline{D}}$: $2x\,\frac{dx}{dt} = 2R\,\frac{dR}{dt}$

$\frac{dR}{dt} = \frac{\cancel{2}x(dx/dt)}{\cancel{2}R}$

S: $\frac{dR}{dt} = \frac{4(1.5)}{\sqrt{41}} = \frac{6}{\sqrt{41}} = \boxed{\frac{6\sqrt{41}}{41}\,m/s}$

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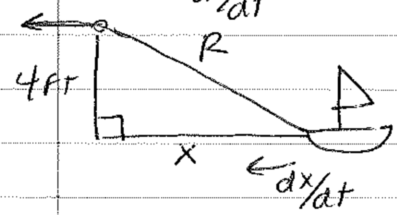
(10)



FIND $\frac{dx}{dt}$ when $R=10$

(IF $R=10$ $x=\sqrt{10^2-4^2}=\sqrt{84}=2\sqrt{21}$)

E: $x^2+4^2=R^2$

D: $2x\,\frac{dx}{dt} = 2R\,\frac{dR}{dt} \rightarrow \frac{dx}{dt} = \frac{R(dR/dt)}{x}$

S: $\frac{dx}{dt} = \frac{10(-2)}{2\sqrt{21}} = \frac{-10}{\sqrt{21}} = \frac{-10\sqrt{21}}{21}$ FT/sec

SO APPROACHING DOCK at $10\sqrt{21}$ FT/sec

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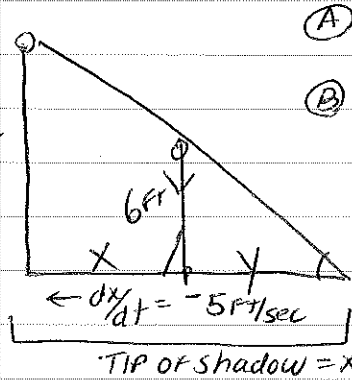
(11)



(A) FIND $\frac{d(x+y)}{dt}$ when $x=10$

(B) FIND $\frac{dy}{dt}$ when $x=10$

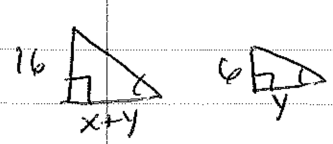
NOTE: IT IS easier to DO PART $\underline{B}$ FIRST!!! SO I WILL.

(B) E: $\frac{16}{x+y} = \frac{6}{y} \rightarrow 16y = 6x+6y$

$10y = 6x$

$y = \frac{3}{5}x$

D: $\frac{dy}{dt} = \frac{3}{5}\frac{dx}{dt}$



S: $\frac{dy}{dt} = \frac{3}{5}(-5) = -3$ FT/sec

SHADOW IS changing length at a rate of -3 FT/sec

(A) tip of shadow $= S = x+y$

$\frac{dS}{dt} = \frac{dx}{dt} + \frac{dy}{dt} = -5 + -3 =$ -8 FT/sec ← rate tip of shadow is moving

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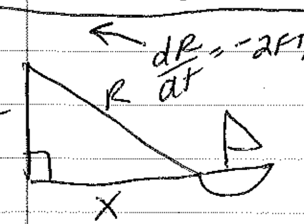
(12)



IF $R=10$ then $x=8$ (3-4-5 Δ)

FIND $\frac{dx}{dt}$ when $R=10$FT

E: $x^2+6^2=R^2$

D: $2x\,\frac{dx}{dt} = 2R\,\frac{dR}{dt} \rightarrow \frac{dx}{dt} = \frac{R(dR/dt)}{x}$

S: $\frac{dx}{dt} = \frac{10(-2)}{8} = \frac{-20}{8} = \frac{-5}{2}$ FT/sec

SO APPROACHING DOCK at $\frac{5}{2}$ FT/sec

3.4RR

FIND $\frac{dx}{dt}$ when $y=1$ AND $\frac{dy}{dt}=2$ (1)

Need to know x ...

(1) E: $x^2+3xy+y^2=1$

$x^2+3x(1)+1^2=1$

$x^2+3x=0$

$x(x+3)=0$

$x=0$ or $x=-3$

D: $2x\frac{dx}{dt}+3x\frac{dy}{dt}+y\left(3\frac{dx}{dt}\right)+2y\frac{dy}{dt}=0$

S: POINT 1 $(0,1)$

$\cancel{2(0)\left(\frac{dx}{dt}\right)}+\cancel{3(0)(2)}+(1)(3)\left(\frac{dx}{dt}\right)+2(1)(2)=0$

$3\,dx/dt=-4$

$\boxed{dx/dt=-4/3}$

POINT 2 $(-3,1)$

$2(-3)\frac{dx}{dt}+3(-3)(2)+1(3)\left(\frac{dx}{dt}\right)+2(1)(2)=0$

$-3\frac{dx}{dt}-14=0$

$\boxed{dx/dt=14/-3}$

Yes... you need both ans!!

(2) E: $y^2=x^2(x+1)$ FIND $\frac{dy}{dt}$ given $\frac{dx}{dt}=a$, $x=1$, $y=-\sqrt{2}$

$y^2=x^3+x^2$

D: $2y\frac{dy}{dt}=3x^2\frac{dx}{dt}+2x\frac{dx}{dt}$

$\frac{dy}{dt}=\frac{(3x^2+2x)(dx/dt)}{2y}$

S: $\frac{dy}{dt}=\frac{(3(1)^2+2(1))(a)}{2(-\sqrt{2})}$

$=\frac{5a}{-2\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)=\boxed{\frac{-5\sqrt{2}\,a}{4}}$

TYPO ON ANS

(3) E: $y=\frac{1}{2}(e^x+e^{-x})$ FIND $\frac{dy}{dt}$ when $\frac{dx}{dt}=1$ and $x=1$

D: $\frac{dy}{dt}=\frac{1}{2}\left(e^x+e^{-x}(-1)\right)\frac{dx}{dt}$ → OR $\frac{dy}{dt}=\frac{1}{2}e^x\frac{dx}{dt}-\frac{1}{2}e^{-x}\frac{dx}{dt}$

S: $\frac{dy}{dt}=\frac{1}{2}(e^1-e^{-1})(1)=\frac{1}{2}\left(e-\frac{1}{e}\right)=\frac{1}{2}\left(\frac{e^2}{e}-\frac{1}{e}\right)=\boxed{\frac{1}{2}\left(\frac{e^2-1}{e}\right)}$

(4) E: $y^2=2x+1$ (POINT IS ON upper HALF) given $\frac{dx}{dt}=\sqrt{2x+1}$ and $x=4$

FIND dy/dt

D: $2y\,dy/dt=2\,dx/dt$

$\frac{dy}{dt}=\frac{\cancel{2}(dx/dt)}{\cancel{2}y}=\frac{dx/dt}{y}=\frac{\sqrt{2x+1}}{y}$

IF $x=4$

$y^2=2(4)+1$

$y^2=9$

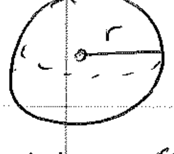
$y=3$ ← NOT -3 b/c upper half of curve

S: $\frac{dy}{dt}=\frac{\sqrt{2(4)+1}}{3}=\frac{\sqrt{9}}{3}=\boxed{1}$

(5) puppies ☺

$\boxed{3.4\ RR}$ (8)

(6)



Ⓐ FIND $\frac{dR}{dt}$

Ⓑ FIND $\frac{dSA}{dt}$

when $r=5$

$\frac{dV}{dt} = 100\,ft^3/min$

E: $V = \frac{4}{3}\pi r^3$

D: $\frac{dV}{dt} = 4\pi r^2\, dr/dt$

$\frac{dr}{dt} = \frac{(dV/dt)}{4\pi r^2}$

S: $\frac{dr}{dt} = \frac{100\pi}{4\pi(25)}$

Ⓐ $\boxed{\frac{dr}{dt} = 1\ FT/MIN}$

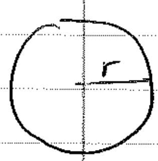
Ⓑ E: $SA = 4\pi r^2$

D: $\frac{dSA}{dt} = 8\pi r\, dr/dt$

S: $\frac{dSA}{dt} = 8\pi(5)(1) = \boxed{40\pi\ FT^2/MIN}$ Ⓑ

---

(7)



$\frac{dV}{dt} = 3\,cm^3/sec$

FIND $\frac{dSA}{dt}$ when $r = 14cm$. (NOTE, AS IN #6, that this requires two rounds of related rates...)

E: $V = \frac{4}{3}\pi r^3$ AND $SA = 4\pi r^2$

D: $\frac{dV}{dt} = 4\pi r^2\, dr/dt$ AND $\frac{dSA}{dt} = 8\pi r\, dr/dt$

$\frac{dr}{dt} = \frac{(dV/dt)}{4\pi r^2}$

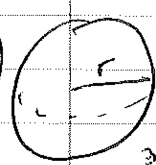
S: $dr/dt = \frac{3}{4\pi(14)^2} = \frac{3}{4\pi \cdot 14 \cdot 14}$ (NOT simplifying b/c IT plan to CANCEL A 14)

$\frac{dSA}{dt} = \cancel{8}^{2}\cancel{\pi}(\cancel{14})\left(\frac{3}{\cancel{4}\cancel{\pi}\cancel{14}\cdot 14}\right)$

$\frac{dSA}{dt} = \frac{6}{14} = \boxed{\frac{3}{7}\ cm^2/sec}$

---

(8)



$\frac{dV}{dt} = 80\,cm^3/sec$

FIND $\frac{dr}{dt}$ when $d = 60cm$ (so $r = 30cm$)

E: $V = \frac{4}{3}\pi r^3$

D: $\frac{dV}{dt} = 4\pi r^2\, dr/dt$

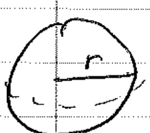
$dr/dt = \frac{(dV/dt)}{4\pi r^2}$

S: $\frac{dR}{dt} = \frac{\cancel{80}^{2}}{\cancel{4}\pi(30\cdot\cancel{30})} = \frac{2}{\pi \cdot 15 \cdot 3}$

$\boxed{\frac{dR}{dt} = \frac{1}{45\pi}\ cm/sec}$

---

(9)



$\frac{dV}{dt} = 100\,\frac{ft^3}{min}$

Ⓐ FIND dr/dt] when

Ⓑ Find dSA/dt] $r=3$

Ⓐ E: $V = \frac{4}{3}\pi r^3$

D: $\frac{dV}{dt} = 4\pi r^2\, dr/dt$

$dr/dt = \frac{(dV/dt)}{4\pi r^2}$

S; $dr/dt = \frac{100}{4\pi(9)} = \boxed{\frac{25}{9\pi}\ FT/MIN}$

Ⓑ E: $SA = 4\pi r^2$

D: $\frac{dSA}{dt} = 8\pi r\, dr/dt$

S: $\frac{dSA}{dt} = 8\cancel{\pi}(\cancel{3})\left(\frac{25}{\cancel{9}_3\cancel{\pi}}\right)$

$\boxed{\frac{dSA}{dt} = \frac{200}{3}\ FT^2/MIN}$

---

(10)

$\frac{dV}{dt} = -2\,cm^3/sec$

FIND dr/dt when $r=5$

E: $V = \frac{4}{3}\pi r^3$

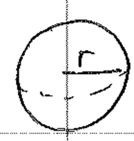
D: $\frac{dV}{dt} = 4\pi r^2\, dr/dt$

$dr/dt = \frac{(dV/dt)}{4\pi r^2}$

S: $\frac{dr}{dt} = \frac{-2}{4\pi(25)} = \frac{-1}{50\pi}\ cm^3/s$

$\boxed{\text{So } \underline{\text{DECREASING}} \text{ at rate of } \frac{1}{50\pi}\ cm^3/s}$

$\boxed{3.4\ RR}$ FIND $\frac{dd}{dt}$ when $d = 10cm$

(11) 

E: $V = \frac{4}{3}\pi r^3$ Not $d = 2r$, $r = d/2$

$\frac{dV}{dt} = 1 \, cm^3/min$

$V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{4}{3}\pi\frac{d^3}{8} = \frac{\pi}{6}d^3$

D: $\frac{dV}{dt} = \frac{\pi}{2}d^2\frac{dd}{dt}$

$dd/dt = \frac{(dV/dt)}{(\frac{\pi}{2}d^2)}$

S: $\frac{dd}{dt} = \frac{-1}{\frac{\pi}{2}(100)} = \boxed{\frac{-1}{50\pi}\ cm/min}$

so decreasing at $1/50\pi$ cm/min

---

(12)  $\frac{dr}{dt} = -15\ cm/min$

FIND $\frac{dV}{dt}$ when $r = 9cm$

E: $V = \frac{4}{3}\pi r^3$

D: $\frac{dV}{dt} = 4\pi r^2 dr/dt$

S: $dV/dt = 4\pi(81)(-15) = -4860\ cm^3/min$

$\boxed{\text{So it is being removed at a rate of } 4860\ cm^3/min}$

---

(13) E: $u = (x-1)^3$ AND $V = (x+1)^3$

D: $\frac{du}{dt} = 3(x-1)^2 dx/dt$ AND $\frac{dV}{dt} = 3(x+1)^2 dx/dt$

S: $6 = 3(x-1)^2(1/2)$ AND $\frac{dV}{dt} = 3(x+1)^2\left(\frac{1}{2}\right) = \frac{3}{2}(x+1)^2$

$12 = 3(x-1)^2$

$4 = (x-1)^2$

$\pm 2 = x - 1$

$2 = x - 1$ or $-2 = x - 1$

$3 = x$ $\quad -1 = x$

SO... IF $x = 3$: $\frac{dV}{dt} = \frac{3}{2}(16) = \boxed{24}$

AND IF $x = -1$: $\frac{dV}{dt} = \frac{3}{2}(0)^2 = \boxed{0}$

(Yes... you need both answers)

---

(14) (A) $\frac{dV}{dt} = 4\pi r^2 dr/dt$ AND $SA = 4\pi r^2$... basically they want you to see that both are $4\pi r^2$

(B) $dV/dt = 7\ ft^3/min$

$\frac{dV/dt}{4\pi r^2} = dr/dt$

$\frac{7}{4\pi(100)} = dr/dt$

$SA = 4\pi r^2$

$\frac{dSA}{dt} = 8\pi r\, dr/dt$

$\frac{dSA}{dt} = 8\pi(10)\left(\frac{7}{4\pi(100)}\right)$

$\frac{dSA}{dt} = \frac{14}{10}\ ft^2/min = \boxed{\frac{7}{5}\ ft^2/min}$

(C) $\frac{dV}{dt}$ is constant.

$\frac{dV}{dt} = 4\pi r^2 dr/dt$

$\boxed{dSA = 8\pi r\, dr/dt}$ ← I think this is all they are asking for... but I'm not sure

(d) $\frac{dSA}{dt} = \frac{1}{3}dV/dt$

$8\pi r\, dr/dt = \frac{1}{3}4\pi r^2 dr/dt$

$8\cancel{\pi} r\, \cancel{dr/dt} = \frac{4}{3}\cancel{\pi} r^2 \cancel{dr/dt}$

$\cancel{\pi}\ \cancel{dr/dt}$

$8r = 4/3\, r^2$

$\frac{4}{3}r^2 - 8r = 0$

$4r\left(\frac{1}{3}r - 2\right) = 0$

$r = 0$ NOT logical

OR

$\frac{1}{3}r = 2$

$\boxed{r = 6}$

TYPO ON ANS

3.4 RR

$d = 6\text{ IN}$ so $r = 3\text{ in}$ $\quad h = 12\text{ IN}$

15 A $V = \pi r^2 h$

$V = \pi(3)^2(12) = 108\pi\text{ IN}^3$

B $V = \pi(3)^2(8) = 72\pi\text{ IN}^3$

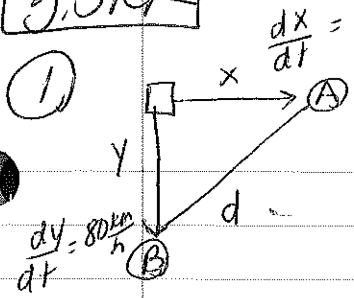
C $V = \pi(3)^2 h$ ← CAN plug in r b/c FOR A cylinder it is constant

E: $V = 9\pi h$

D: $\frac{dV}{dt} = 9\pi\, dh/dt \qquad dV/dt = 5\text{ IN}^3/\text{min} \qquad h = 8\text{ in}$

$dh/dt = \frac{(dV/dt)}{9\pi} = \boxed{\frac{5}{9\pi}\text{ IN/MIN}}$

(1)



FIND $\frac{dd}{dt}$ After 4 hours

E: $x^2 + y^2 = d^2$

D: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2d\frac{dd}{dt}$

S: After 4 hours $400 = x$, $320 = y$, $d = \sqrt{400^2 + 320^2} = \sqrt{262400} = 80\sqrt{41}$

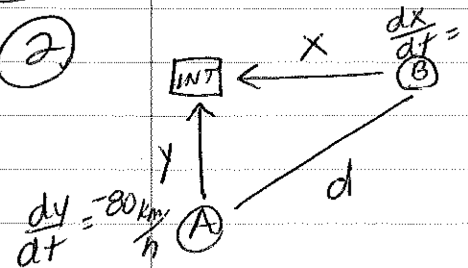
$$\frac{dd}{dt} = \frac{(x\,dx/dt + y\,dy/dt)}{d}$$

$$\frac{dd}{dt} = \frac{400(100) + 80(320)}{80\sqrt{41}} = \frac{40000 + 25600}{80\sqrt{41}}$$

$$dd/dt = \frac{820}{\sqrt{41}} = \frac{820\sqrt{41}}{41} = 20\sqrt{41}$$

$$\boxed{\frac{dd}{dt} = 20\sqrt{41}\ \text{km/hr}}$$

(2)



FIND $\frac{dd}{dt}$ when ~~x~~ $y = .4$ km AND $x = .7$ km

$d = \sqrt{.7^2 + .4^2} = \sqrt{.65} = \frac{\sqrt{65}}{10}$

E: $x^2 + y^2 = d^2$

D: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2d\,dd/dt$

$$dd/dt = \frac{x(dx/dt) + y(dy/dt)}{d}$$

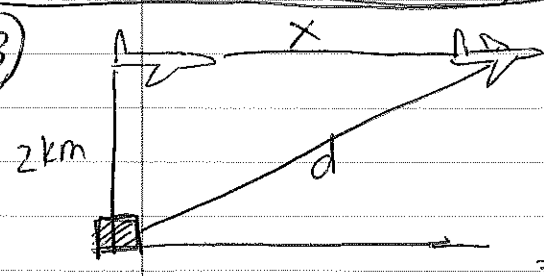
S: $$\frac{dd}{dt} = \frac{.7(-110) + (.4)(-80)}{\left(\frac{\sqrt{65}}{10}\right)} = \frac{7(-110) + 4(-80)}{\sqrt{65}}$$

$$\frac{dd}{dt} = \frac{-770 - 320}{\sqrt{65}} = \frac{-1090\sqrt{65}}{65}$$

$$\boxed{\frac{dd}{dt} = \frac{-218\sqrt{65}}{13}\ \text{km/hr}}$$

So approaching each other at a rate of $\frac{218\sqrt{65}}{13}$ km/hr

(3)



$\rightarrow dx/dt = 700$ km/h

Find dd/dt when $d = 6$ km

E: $x^2 + 2^2 = d^2$

D: $2x\,dx/dt = 2d\,dd/dt \rightarrow \frac{dd}{dt} = \frac{x(dx/dt)}{d}$

when $d = 6$

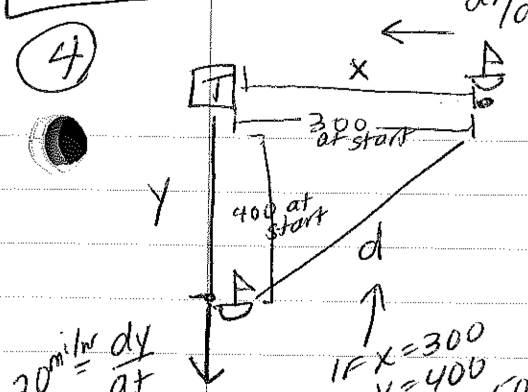
$x = \sqrt{6^2 - 2^2} = \sqrt{32} = 4\sqrt{2}$

S: $$\frac{dd}{dt} = \frac{4\sqrt{2}(700)}{6} = \boxed{\frac{1400\sqrt{2}}{3}\ \text{km/h}}$$

3.5 RR

(12) 1

(4)



IF $x = 300$
$y = 400$
then $d = 500$
(3-4-5 Δ)

E: $x^2 + y^2 = d^2$

D: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2d\frac{dd}{dt}$

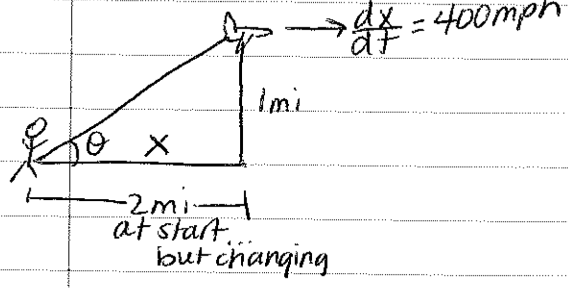
S: $\frac{dd}{dt} = \frac{x(dx/dt) + y(dy/dt)}{d}$

$\frac{dd}{dt} = \frac{3\cancel{00}(-15) + (4\cancel{00})(20)}{5\cancel{00}}$

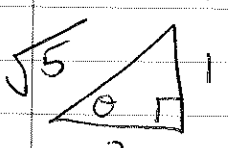
$= \frac{-45 + 80}{5} = \frac{35}{5} = \boxed{7 \text{ mi/hr}}$

---

(5)



FIND $d\theta/dt$ when $x = 2$



so $\sec\theta = \frac{\sqrt{5}}{2}$
OR $\cos\theta = 2/\sqrt{5}$

E: $\tan\theta = \frac{1}{x} = x^{-1}$

D: $\sec^2\theta \frac{d\theta}{dt} = -x^{-2} dx/dt$

$\sec^2\theta(d\theta/dt) = \frac{-1}{x^2}(dx/dt)$

$\left(\frac{1}{\cos^2\theta}\right)\frac{d\theta}{dt} = \frac{-1}{x^2} dx/dt$

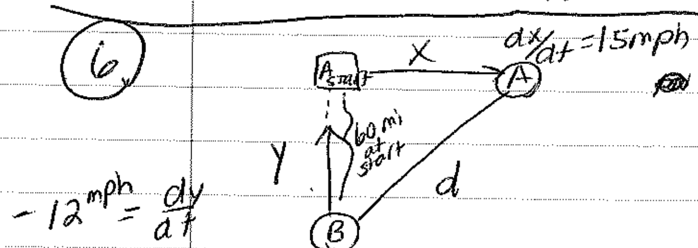
$\frac{d\theta}{dt} = \cos^2\theta\left(\frac{-1}{x^2}\right) dx/dt$

S: $\frac{d\theta}{dt} = \left(\frac{2}{\sqrt{5}}\right)^2\left(\frac{-1}{4}\right)(400)$

$= \frac{\cancel{4}}{5}\left(\frac{-1}{\cancel{4}}\right)(400) = \boxed{-80 \text{ rad/hr}}$

---

(6)



After 4hrs:
$x = 4(15) = 60$ miles
$y = 60 - 4(12) = 12$ miles
$d = \sqrt{60^2 + 12^2} = 12\sqrt{26}$

E: $x^2 + y^2 = d^2$

D: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2d\frac{dd}{dt}$

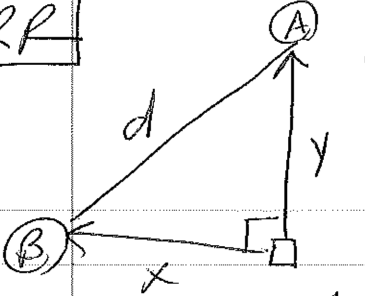
$\frac{dd}{dt} = \frac{x(dx/dt) + y(dy/dt)}{d}$

S: $\frac{dd}{dt} = \frac{60(15) + 12(-12)}{12\sqrt{26}}$

$\frac{dd}{dt} = \frac{75 - 12}{\sqrt{26}} = \boxed{\frac{63\sqrt{26}}{26} \text{ mph}}$

3.5RR

7



$dy/dt = 10$ knots

$dx/dt = 30$ knots

IF SHIP A HAS been out 5hrs

~~Y~~ $= 50$ N.M. AND

Ship B has been out

2 hrs ... so $X = 60$ N.M

so $d = \sqrt{60^2+50^2} = 10\sqrt{61}$

E: $x^2+y^2=d^2$

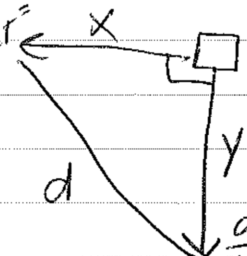
D: $2x\,dx/dt + 2y\,dy/dt = 2d\,dd/dt$

$$dd/dt = \frac{x(dx/dt)+y(dy/dt)}{d}$$

S: $$\frac{dd}{dt} = \frac{60(3\cancel{0})+(5\cancel{0})(10)}{1\cancel{0}\sqrt{61}}$$

$$\frac{dd}{dt} = \frac{180+50}{\sqrt{61}} = \boxed{\frac{230\sqrt{61}}{61}\text{ KNOTS}}$$

8



$dx/dt = 25$mph

$\frac{dy}{dt} = 60$mph

two hrs later:

$x = 50$mi

$y = 120$mi

$d = 130$mi (5-12-13 △)

E: $x^2+y^2=d^2$

D: $2x\frac{dx}{dt} + 2y\,dy/dt = 2d\,dd/dt$

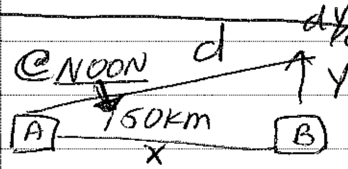
$$dd/dt = \frac{x(dx/dt)+y(dy/dt)}{d}$$

S: $$\frac{dd}{dt} = \frac{5\cancel{0}(25)+12\cancel{0}(60)}{13\cancel{0}}$$

$$\frac{dd}{dt} = \frac{125+720}{13} = \frac{845}{13} = 65$$

$\boxed{dd/dt = 65\text{mph}}$

9

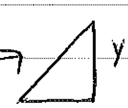


$dy/dt = 25$ km/hr

$dx/dt = -35$ km/hr

FIND

dd/dt at 4pm



$x = 150-140$

$= 10$km

$d = \sqrt{100^2-10^2}$

$d = 30\sqrt{11}$

E: $x^2+y^2=d^2$

D: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2d\frac{dd}{dt}$

$$\frac{dd}{dt} = \frac{x\frac{dx}{dt}+y\,dy/dt}{d}$$

S: $$\frac{dd}{dt} = \frac{10(-35)+(100)(25)}{30\sqrt{11}} = \frac{-35+250}{3\sqrt{11}}$$

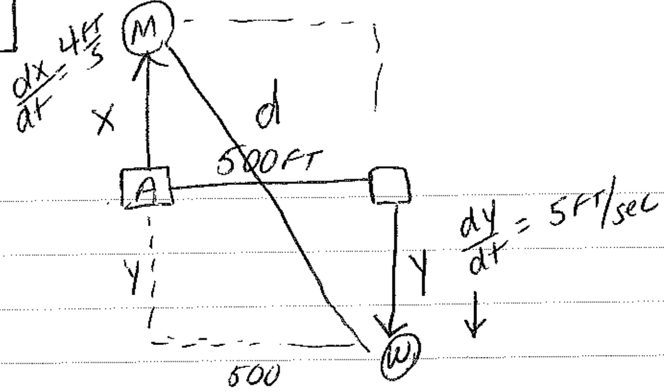
$$\boxed{\frac{dd}{dt} = \frac{215}{3\sqrt{11}}\text{ km/hr} = 21.608\text{ km/hr}}$$

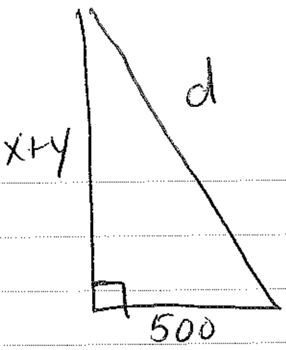
(TYPO ON ANS)

3.5 RR

(14)

(10) TRICKY





FIND dd/dt ... 15 min later

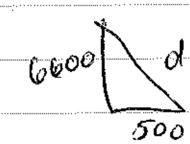
$15\text{ min} = 15(60) = 900\text{ sec}$

MAN HAS WALKED FOR 900 SEC

SO $x = 4(900) = 3600$ FT

WOMAN HAS WALKED FOR 10 min = 600 sec

SO $y = 5(600) = 3000$ FT



$d = \sqrt{6600^2 + 500^2}$

$d = 100\sqrt{4381}$

E: $(x+y)^2 + 500^2 = d^2$

D: $2(x+y)\left(\frac{dx}{dt} + \frac{dy}{dt}\right) = 2d\frac{dd}{dt}$

$$\frac{dd}{dt} = \frac{(x+y)\left(\frac{dx}{dt} + \frac{dy}{dt}\right)}{d}$$

S: $$\frac{dd}{dt} = \frac{(66\cancel{0}\cancel{0})(4+5)}{10\cancel{0}\sqrt{4381}} = \frac{594}{\sqrt{4381}}$$

$$\boxed{\frac{dd}{dt} = \frac{594\sqrt{4381}}{4381}\,ft/sec \approx 8.897\,ft/s}$$

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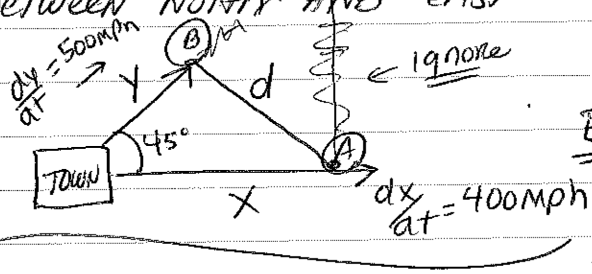
(11) Assume "Northeast" means at a 45° angle between "North" and "east"

TRICKY b/c it's NOT A right Δ... use LAW OF COSINES



E: $d^2 = x^2 + y^2 - 2xy\cos 45°$

$d^2 = x^2 + y^2 - \sqrt{2}xy$

D: $2d\frac{dd}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} - \sqrt{2}x\frac{dy}{dt} + (-\sqrt{2})y\frac{dx}{dt}$

D: $2d\frac{dd}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} - \sqrt{2}x\frac{dy}{dt} + y(-\sqrt{2})\frac{dx}{dt}$

$$\frac{dd}{dt} = \frac{2x\frac{dx}{dt} + 2y\frac{dy}{dt} - \sqrt{2}x\frac{dy}{dt} - \sqrt{2}y\frac{dx}{dt}}{2d}$$

S: plane A was flying for 1.5 hrs ... so $x = 1.5(400) = 600$ miles

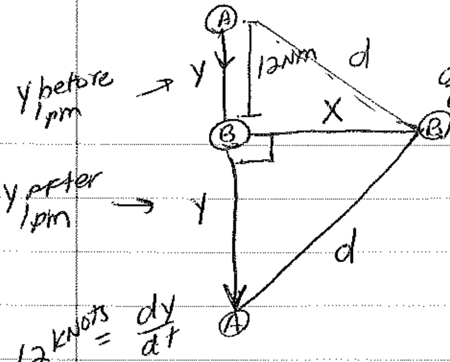
plane B was flying for 1 hr ... so $y = 1(500) = 500$ miles

$d = \sqrt{600^2 + 500^2 - 2(600)(500)(\sqrt{2}/2)} = 430.970_{(1)}$ STO A

$$\frac{dd}{dt} = \frac{2(600)(400) + 2(500)(500) - \sqrt{2}(600)(500) - \sqrt{2}(500)(400)}{2d} = \boxed{316.602_{(3)}\text{ mph}}$$

3.5 RR

(12)



$\frac{dx}{dt} = 8$ KNOTS

12 KNOTS $= \frac{dy}{dt}$

NOTE: FROM NOON to 1pm, $dy/dt = -12$ KNOTS but After 1pm (and at 1pm) $dy/dt = +12$ KNOTS

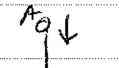
(15)

E: $x^2 + y^2 = d^2$

D: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2d\frac{dd}{dt}$

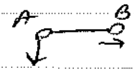
$$\frac{dd}{dt} = \frac{\cancel{2}x\,dx/dt + \cancel{2}y\,dy/dt}{\cancel{2}d}$$

(A) at NOON: $y = 12$ NM $x = 0$ NM $d = 12$ NM



$$\frac{dd}{dt} = \frac{0 + 12(-12)}{12} = \boxed{-12 \text{ knots}}$$

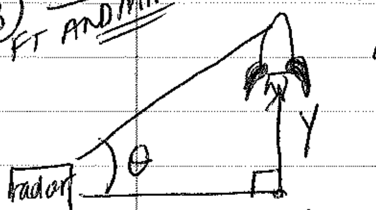
(B) at 1pm: $x = 8$ NM $y = 0$ NM $d = 8$ NM



$$\frac{dd}{dt} = \frac{8(8) + 0(12)}{8} = \boxed{8 \text{ knots}}$$

(C) No, they are never able to see one another... the closest they ever get is 8NM

(13) TRICKY FT AND Miles



$.5$ mi $= 2640$ FT

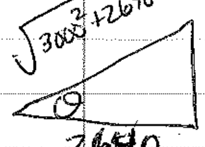
FIND $\frac{d\theta}{dt}$ when $y = 3000$ FT AND $\frac{dy}{dt} = 500$ ft/sec

E: $\tan\theta = \frac{y}{2640}$

D: $\sec^2\theta \frac{d\theta}{dt} = \frac{dy}{dt}\left(\frac{1}{2640}\right)$

$$\frac{d\theta}{dt} = \cos^2\theta \frac{dy}{dt}\left(\frac{1}{2640}\right)$$

S: $\sqrt{3000^2 + 2640^2} = 3996.198 \xrightarrow{} $ STO A



$$\cos^2\theta = \frac{2640^2}{15969600} = \frac{6969600}{15969600}$$

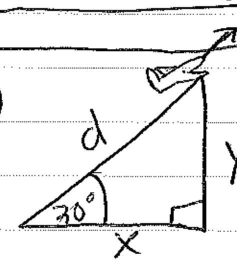
$$\cos^2\theta = \frac{484}{1109}$$

S: $\frac{d\theta}{dt} = \frac{484}{1109}(500)\left(\frac{1}{2640}\right)$

$$= \frac{242000}{2927760} = \frac{275}{3327} = .08\overline{2}\ (3)$$

$$\boxed{\frac{d\theta}{dt} = .08\overline{2} \text{ rad/sec}\ (3)}$$

(14)



FIND dy/dt given $\frac{dd}{dt} = 500$ mph

E: $\sin 30° = \frac{y}{d}$

$\frac{1}{2} = y/d$

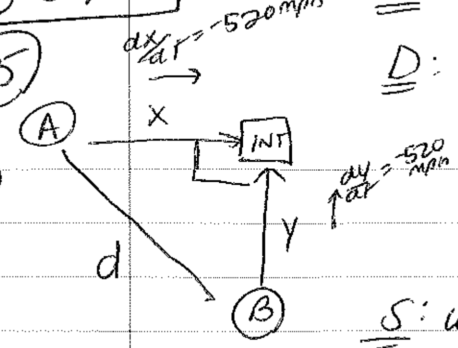
$d = 2y$

D: $\frac{dd}{dt} = 2\frac{dy}{dt} \rightarrow \frac{1}{2}\frac{dd}{dt} = dy/dt$

S: $\boxed{dy/dt = 250 \text{mph}}$

$\boxed{3.5\ RR}$ 16

15



E: $x^2+y^2=d^2$

D: $2x\frac{dx}{dt}+2y\frac{dy}{dt}=2d\frac{dd}{dt}$

$$\frac{dd}{dt}=\frac{x\,dx/dt+y\,dy/dt}{d}$$

S: when $x=5$ and $y=12$... then $d=13$ (5-12-13 Δ)

$$\frac{dd}{dt}=\frac{5(-520)+12(-520)}{13}=\frac{7(-520)}{13}=-280\text{ mph}$$

$\boxed{\text{closing at a rate of 280 mph}}$

AT ANY TIME

$$\boxed{\frac{dd}{dt}=\frac{-520(x+y)}{d}\text{ mph}}$$

given: $\boxed{dV/dt = 1\text{ volt/sec}}$ ← (A)

$\boxed{dI/dt = -1/3\text{ amp/sec}}$ ← (B)

both given

16 E: $V=IR$

D: $\boxed{\frac{dV}{dt}=I\frac{dR}{dt}+R\frac{dI}{dt}}$ ← (C)

Later, given $V=12$, $I=2$

$V=IR$

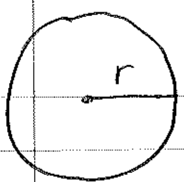
$12=2R$

$R=6$

$$\frac{\frac{dV}{dt}-R\frac{dI}{dt}}{I}=\frac{dR}{dt}$$

S: $\frac{dR}{dt}=\frac{1-6(-\frac{1}{3})}{2}=\frac{1+2}{2}=\boxed{+\frac{3}{2}\ \frac{\text{ohms}}{\text{sec}}}$ ← (D) $\boxed{\text{it's \sout{decreasing} increasing}}$

(1)



$dr/dt = .8$ m/s

D: $\frac{dA}{dt} = 2\pi r \, dr/dt$

S: $\frac{dA}{dt} = 2\pi(6)(.8) = \boxed{9.6\pi \text{ m}^2/\text{sec}}$

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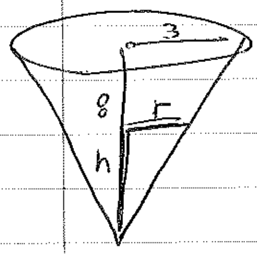
(2)



$\frac{8}{3} = \frac{h}{r}$

$8r = 3h$

$r = \frac{3}{8}h$

E: $V = \frac{1}{3}\pi r^2 h$ replace r with h-stuff

$V = \frac{1}{3}\pi\left(\frac{3h}{8}\right)^2 h = \frac{3\pi}{64} h^3$

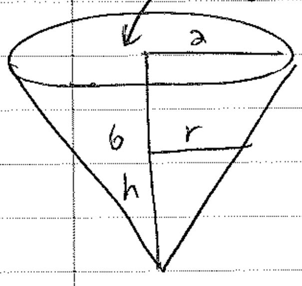
D: $\frac{dV}{dt} = \frac{9\pi}{64} h^2 \, dh/dt$

$\frac{dh}{dt} = \frac{64}{9\pi h^2} \frac{dV}{dt}$

(3)

$dV/dt = 5 \text{ m}^3/\text{min}$

S: $\frac{dh}{dt} = \frac{64}{9\pi(25)}(3) = \boxed{\frac{64}{75\pi} \text{ m/min}}$



$\frac{6}{2} = \frac{h}{r}$

$3r = h$

E: $V = \frac{1}{3}\pi r^2 h$ replace h with r-stuff

$V = \frac{1}{3}\pi r^2(3r) = \pi r^3$

D: $\frac{dV}{dt} = 3\pi r^2 \, dr/dt \longrightarrow \frac{dr}{dt} = \frac{1}{3\pi r^2}\left(\frac{dV}{dt}\right)$

S: $\frac{dr}{dt} = \frac{1}{3\pi(1.2)^2}(5) = \boxed{\frac{5}{4.32\pi} \text{ m/min}}$ TYPO ON worksheet FOR ANS

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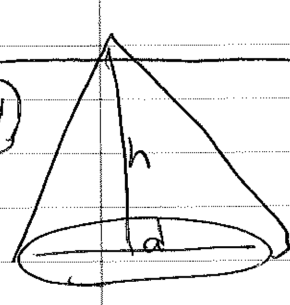
(4)



$d = 2h$

$2r = 2h$

$r = h$

E: $V = \frac{1}{3}\pi r^2 h$ replace r with h-stuff

$V = \frac{1}{3}\pi h^2 h = \frac{1}{3}\pi h^3$

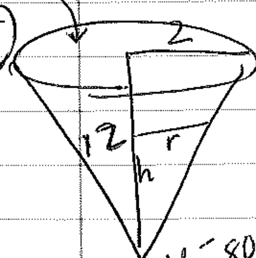
D: $\frac{dV}{dt} = \pi h^2 \, dh/dt \longrightarrow \frac{dh}{dt} = \frac{1}{\pi h^2} dV/dt$

S: $\frac{dh}{dt} = \frac{1}{\pi(144)}(25) = \boxed{\frac{25}{144\pi} \text{ FT/min}}$

---

(5)

$da/dt = ??$



$\frac{dl}{dt} = 8000 \text{ m}^3/\text{min}$

$\frac{12}{2} = \frac{h}{r}$

$6r = h$

$r = h/6$

E: $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{6}\right)^2 h = \frac{\pi}{108} h^3$

D: $\frac{dV}{dt} = \frac{\pi}{36} h^2 \frac{dh}{dt}$

S: $\frac{dV}{dt} = \frac{\pi}{36}(49)(.4) = \frac{4.9\pi}{9}$

$$\boxed{\frac{dA}{dt} = \left(\frac{4.9\pi}{9} + 8000\right) \text{ m}^3/\text{min}}$$

TYPO ON original worksheet for ans

$\frac{dV}{dt} = \frac{da}{dt} - 8000$

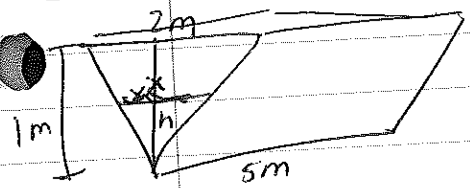
(AMT IN) (AMT OUT)

$\frac{da}{dt} = \frac{dV}{dt} + 8000$

3.8 RR

$E: V = \frac{1}{2} x h \cdot 5$ $\quad \frac{1}{2} = \frac{h}{x}$ $\quad x = 2h$

(6)



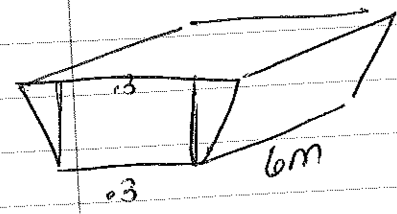
$V = \frac{1}{2}(2h)(h)(5) = 5h^2$

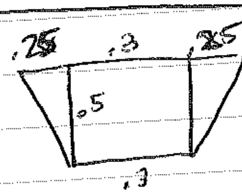
$D: \frac{dV}{dt} = 10h \, dh/dt \longrightarrow \frac{dh}{dt} = \frac{1}{10h}\left(\frac{dV}{dt}\right)$

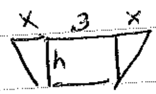
$S: \frac{dh}{dt} = \frac{1}{10(.4)}(60) = \frac{60}{4} = \boxed{15 \text{ m/sec}}$

---

(7)







$E: V = \underbrace{\frac{1}{2}(h)(.3 + .3 + x + x)}_{\text{Area of base}} \cdot \underset{\text{length of prism}}{6}$

$V = 3h(.6 + 2x)$

$V = 3h(.6 + h)$

$V = 1.8h + 3h^2$

$\frac{.5}{.25} = \frac{h}{x} = 2$

$h = 2x$

$h/2 = x$

$D: \frac{dV}{dt} = 1.8\frac{dh}{dt} + 6h\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{(dV/dt)}{(1.8 + 6h)}$

$S: \frac{dh}{dt} = \frac{.3}{1.8 + 6(.25)}$

$\frac{dh}{dt} = \frac{.3}{1.8 + 1.5} = \frac{.3}{3.3} = \frac{1}{11}$

$\boxed{dh/dt = 1/11 \text{ m/min}}$

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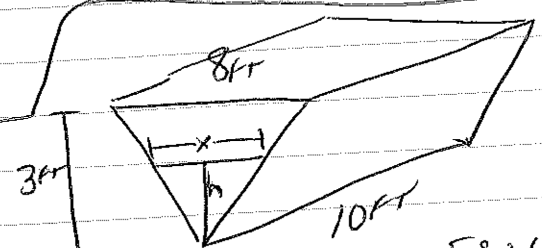
(8)



$\frac{h}{x} = \frac{3}{8}$

$8h = 3x \rightarrow x = \frac{8}{3}h$

$E: V = \frac{1}{2} x h \cdot 10 = 5xh$

$V = 5\left(\frac{8}{3}h\right)h = \frac{40}{3}h^2$

$D: \frac{dV}{dt} = \frac{80}{3}h \, dh/dt$

$dh/dt = (dV/dt)\left(\frac{3}{80h}\right)$

$S: \frac{dh}{dt} = (2)\left(\frac{3}{80(2)}\right) = \boxed{\frac{3}{80} \frac{\text{FT}}{\text{MIN}}}$

OR $.0375 \frac{\text{FT}}{\text{MIN}}$ or $.45$ IN/min

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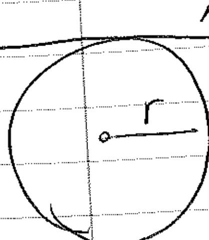
(9)



$E: A = \pi r^2$

$D: \frac{dA}{dt} = 2\pi r \, dr/dt$

when $A = 9 \text{ km}^2$

$9 = \pi r^2$

$\sqrt{\frac{9}{\pi}} = r$

$S: 6 = 2\pi\sqrt{\frac{9}{\pi}}(dr/dt)$

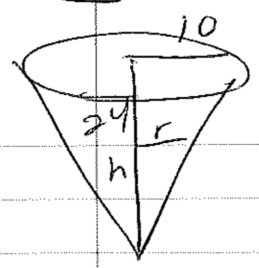
$6 = 2\pi \cdot \frac{3}{\sqrt{\pi}} \, dr/dt$

$\frac{6}{2\pi} = \frac{3}{\sqrt{\pi}} \frac{dr}{dt}$

$\frac{6}{2\pi} \cdot \frac{\sqrt{\pi}}{3} = dr/dt = \boxed{\frac{1}{\sqrt{\pi}} \frac{\text{km}}{\text{hr}}}$

3.8RR

10



$\frac{24}{10} = \frac{h}{r}$

$\frac{12}{5} = \frac{h}{r}$

$12r = 5h$

$r = \frac{5}{12}h$

E: $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{5}{12}h\right)^2 h$

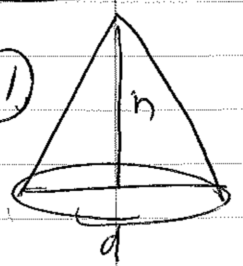
$V = \frac{25}{432}\pi h^3$

D: $\frac{dV}{dt} = \frac{25\pi}{144}h^2\frac{dh}{dt}$

$\frac{dh}{dt} = \left(\frac{dV}{dt}\right)\left(\frac{144}{25\pi h^2}\right)$

S: $\frac{dh}{dt} = 20 \cdot \frac{\overset{12 \cdot 12}{144}}{25\pi(16)^2} = \boxed{\frac{9}{20\pi}\ \text{FT/MIN}}$

11



$h = d$

$h = 2r$

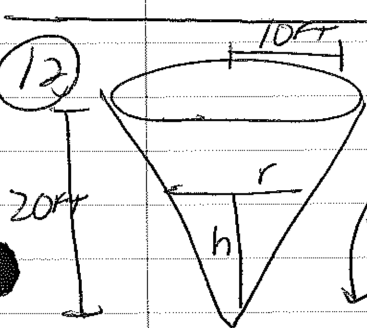
$\frac{h}{2} = r$

E: $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{\pi}{12}h^3$

D: $dV/dt = \frac{\pi}{4}h^2\,dh/dt$

S: $\frac{dV}{dt} = \frac{\pi}{4}(10)^2(5) = \frac{500\pi}{4} = \boxed{125\pi\ \text{FT}^3/\text{MIN}}$

12



$\frac{20}{10} = \frac{h}{r}$

$2 = h/r$

$2r = h$

$r = h/2$

TRICKY... WANT $\frac{dA}{dt}$...

E: $A = \pi r^2$ AND $V = \frac{1}{3}\pi r^2 h$

D: $\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

so I need dr/dt

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$V = \frac{1}{3}\frac{\pi}{4}h^3$

$\frac{dV}{dt} = \frac{\pi}{4}h^2\,dh/dt$

$5 = \frac{\pi}{4}(64)\,dh/dt$

$5 = 16\pi\,dh/dt$

$dh/dt = 5/16\pi$

$2r = h$

$2\frac{dr}{dt} = dh/dt$

$\frac{dr}{dt} = \frac{1}{2}\,dh/dt$

$\frac{dr}{dt} = \frac{1}{2}\left(\frac{5}{16\pi}\right) = \frac{5}{32\pi}$

$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

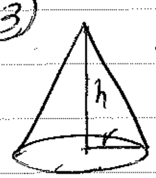
IF $h = 8$

$r = h/2 = 4$

$\frac{dA}{dt} = 2\pi(4)\left(\frac{5}{32\pi}\right)$

$\boxed{\frac{dA}{dt} = \frac{5}{4}\ \text{FT}^2/\text{MIN}}$

13



$h = \frac{2}{3}d$

~~$h = \frac{2}{3}(2r)$~~

~~$h = \frac{4}{3}r$~~

$\frac{d}{2} = r$

$dV/dt = 3\ \text{FT}^3/\text{sec}$

FIND dd/dt

E: $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{d}{2}\right)^2\left(\frac{2}{3}d\right)$

$V = \frac{1}{3}\pi\frac{d^2}{4}\cdot\frac{2}{3}d = \frac{\pi}{18}d^3$

D: $\frac{dV}{dt} = \frac{\pi}{6}d^2\frac{dd}{dt}$

$\frac{dd}{dt} = \left(\frac{dV}{dt}\right)\left(\frac{6}{\pi d^2}\right)$

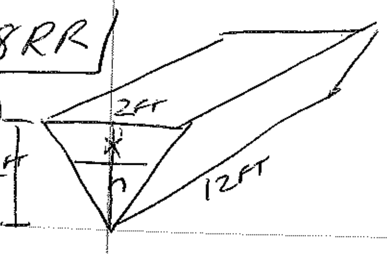
S: $\frac{dd}{dt} = (3)\frac{6}{\pi(36)}$

$\boxed{\frac{dd}{dt} = \frac{1}{2\pi}\ \text{FT/sec}}$

* INCORRECT ANSWER ON ANSWER SHEET

3.8RR

(14)



$h = x$

$\underline{E}$: $V = \frac{1}{2}xh \cdot 12 = 6xh = 6h^2$

$\underline{D}$: $\frac{dV}{dt} = 12h\, dh/dt$

$\frac{dh}{dt} = \left(\frac{dV}{dt}\right)\left(\frac{1}{12h}\right)$

$\underline{S}$: $\frac{dh}{dt} = (1)\frac{1}{(12(1.5))} = \boxed{\frac{1}{18}\ \text{FT/MIN}}$

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(15) IT IS A constant Amount of oil ... So V is A constant 1 cm³.

$V = 1\,cm^3 = \pi r^2 h$

$\underline{E}$: $1 = \pi r^2 h$

$\underline{D}$: $0 = \pi r^2 \frac{dh}{dt} + h \cdot 2\pi r\, dr/dt$

when $r = 8m$

$1 = \pi(64)h$

$\frac{1}{64\pi} = h$

$\underline{S}$: $\frac{-\pi r^2 dh/dt}{2\pi r h} = dr/dt$

$\frac{-\cancel{\pi}(\cancel{64})^{8}(-.1)}{2\cancel{\pi}(\cancel{8})(\frac{1}{64\pi})} = dr/dt$

$\frac{dr}{dt} = \frac{\left(\frac{8}{10}\right)}{\left(\frac{1}{32\pi}\right)} = \frac{8}{10} \cdot \frac{32\pi}{1} = \frac{4(32\pi)}{5}$

$\boxed{\frac{dr}{dt} = \frac{128\pi}{5}\ cm/hr = 25.6\pi\ cm/hr}$

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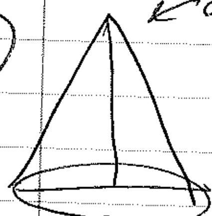
(16)



$dV/dt = 10\ FT^3/min$

$r = 2h$

Find dh/dt when $h = 8FT$

$\underline{E}$: $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(2h)^2 h = \frac{4}{3}\pi h^3$

$\underline{D}$: $\frac{dV}{dt} = 4\pi h^2\, dh/dt$

$\frac{dV}{dt}\left(\frac{1}{4\pi h^2}\right) = \frac{dh}{dt}$

$\underline{S}$: $\frac{dh}{dt} = (10)\frac{1}{4\pi(64)} = \boxed{\frac{5}{128\pi}\ \text{FT/MIN}}$ ← INCORRECT ANS ON the other ANSWER SHEET

COOL BEANS??