AP BC Calculus (APBC) S.Hogan
Midterm REVIEW

This review is identical to the midterm in format.

Name:				
DO NOT WRITE IN ACTUAL SCORE BOXES!!!!	**Section IA Actual**	**Section IB Actual**	**Section IIA Actual**	**Section IIB Actual**

Actual Raw Score Calculation: (HOGAN'S USE ONLY)

0.75(_____ + _____) + _____ + _____ = _____

0.75 (Multiple-Choice Total) + (Section IIA) + (Section IIB) = RAW SCORE

Score Conversions:

RAW RANGE	AP SCORE	GRADEBOOK SCORE
[43, 54]	5	100/100
[33, 43)	4	90/100
[25, 33)	3	80/100
[20, 25)	2	70/100
[0, 19)	1	65/100

Exam Format:

- **Section I:**
  - **Part A: Non-Calculator Multiple-Choice (12 questions—24 minutes)**
  - **Part B: Calculator Multiple-Choice (6 questions—18 minutes)**
- **Section II:**
  - **Part A: Calculator Free-Response (1 question—15 minutes)**
  - **Part B: Non-Calculator Free-Response (2 questions—30 minutes)**

Section I Answer Sheet:

SECTION IA	
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SECTION IA	
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SECTION IB	
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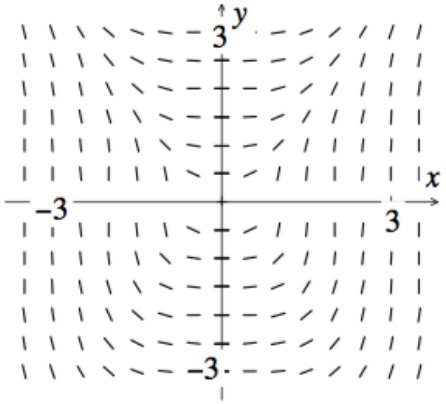
SECTION IA: Non-Calculator Multiple-Choice (12 questions—24 minutes)

1



Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx}=\frac{x}{y}$

(B) $\frac{dy}{dx}=\frac{x^2}{y^2}$

(C) $\frac{dy}{dx}=\frac{x^3}{y}$

(D) $\frac{dy}{dx}=\frac{x^2}{y}$

(E) $\frac{dy}{dx}=\frac{x^3}{y^2}$

2

$$\int \frac{2x}{(x+2)(x+1)}dx=$$

(A) $\ln|x+2|+\ln|x+1|+C$

(B) $\ln|x+2|+\ln|x+1|-3x+C$

(C) $-4\ln|x+2|+2\ln|x+1|+C$

(D) $4\ln|x+2|-2\ln|x+1|+C$

(E) $2\ln|x|+\frac{2}{3}x+\frac{1}{2}x^2+C$

3

$$\int x^2 \cos\left(x^3\right) dx =$$

(A) $-\frac{1}{3}\sin\left(x^3\right)+C$

(B) $\frac{1}{3}\sin\left(x^3\right)+C$

(C) $-\frac{x^3}{3}\sin\left(x^3\right)+C$

(D) $\frac{x^3}{3}\sin\left(x^3\right)+C$

(E) $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right)+C$

4

$$\frac{d}{dx}\left(\int_0^{x^3} \ln(t^2+1)dt\right) =$$

(A) $\frac{2x^3}{x^6+1}$

(B) $\frac{3x^2}{x^6+1}$

(C) $\ln(x^6+1)$

(D) $2x^3\ln(x^6+1)$

(E) $3x^2\ln(x^6+1)$

5

A curve has slope $2x+3$ at each point (x,y) on the curve. Which of the following is an equation for this curve if it passes through the point $(1,2)$?

(A) $y=5x-3$

(B) $y=x^2+1$

(C) $y=x^2+3x$

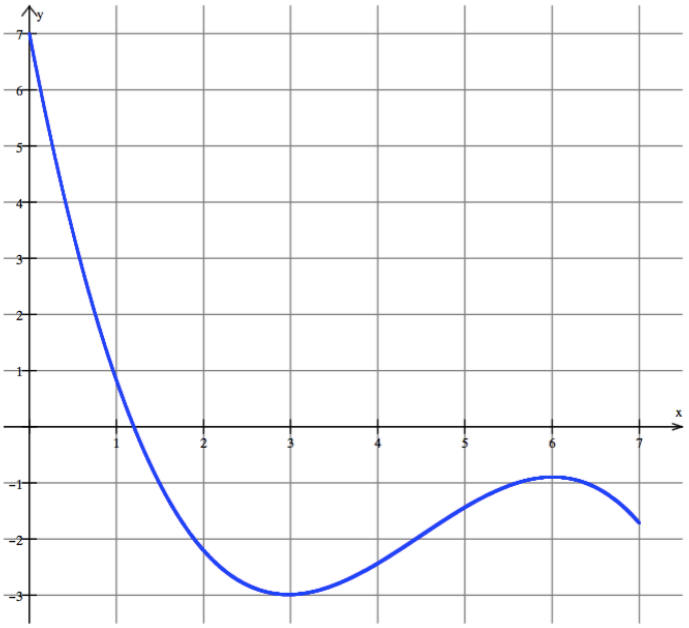
(D) $y=x^2+3x-2$

(E) $y=2x^2+3x-3$

6	If $y = \sin(3x)$ then $\frac{dy}{dx} =$ (A) $-3\cos(3x)$ (B) $-\cos(3x)$ (C) $-\frac{1}{3}\cos(3x)$ (D) $\cos(3x)$ (E) $3\cos(3x)$
7	The rate of change of the volume, V, of water in a tank with respect to time, t, is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship? (A) $V(t) = k\sqrt{t}$ (B) $V(t) = k\sqrt{V}$ (C) $\frac{dV}{dt} = k\sqrt{t}$ (D) $\frac{dV}{dt} = \frac{k}{\sqrt{V}}$ (E) $\frac{dV}{dt} = k\sqrt{V}$
8	$\lim_{x\to 0} \frac{e^x - \cos x - 2x}{x^2 - 2x}$ is (A) $-\frac{1}{2}$ (B) 0 (C) $\frac{1}{2}$ (D) 1 (E) *nonexistent*

9

Graph of f (x)



Graph of f

) The graph of the function f shown in the figure above has horizontal tangents at $x=3$ and $x=6$. If $g(x)=\int_0^{2x} f(t)dt$, what is the value of $g'(3)$?

(A) 0
(B) –1
(C) –2
(D) –3
(E) –6

10

The slope of the tangent to the curve $y^3x + y^2x^2 = 6$ at $(2, 1)$ is

(A) $-\frac{3}{2}$

(B) -1

(C) $-\frac{5}{14}$

(D) $-\frac{3}{14}$

(E) 0

11 If $f(x) = \sin 2x \cos 3x$ and k is an odd integer, then $f'(k\pi) =$

(A) -5 (B) -2 (C) -1 (D) 1 (E) 5

12

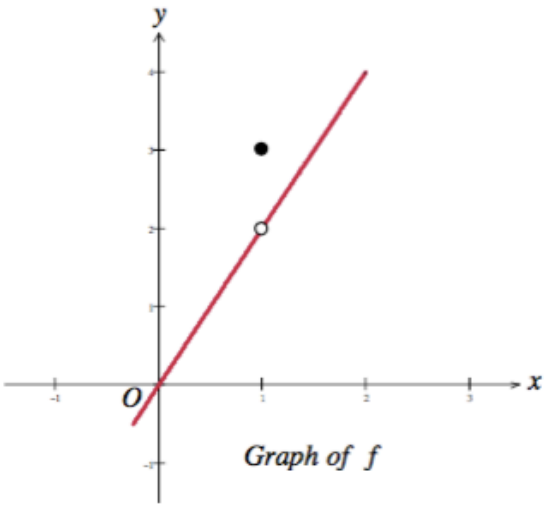
x	$g(x)$	$g'(x)$
1	3	4
2	8	3

If $g(x)$ and $g'(x)$ have the values shown in the table above, and $f(x) = g^2(x)$, then $f'(2) =$

(A) 12 (B) 16 (C) 23 (D) 24 (E) 48

SECTION IB: Calculator Multiple-Choice (6 questions—18 minutes)

13



Graph of f

) The graph of the function f is shown in the figure above. The value of $\lim_{x\to 1} \sin\big(f(x)\big)$ is

(A) 0.909
(B) 0.841
(C) 0.141
(D) –0.416
(E) nonexistent

14

) The height h, in meters, of an object at time t is given by $h(t) = 24t + 24t^{3/2} - 16t^2$. What is the height of the object at the instant when it reaches its maximum upward velocity?

(A) 2.545 meters
(B) 10.263 meters
(C) 34.125 meters
(D) 54.889 meters
(E) 89.005 meters

15

x	0	1	2	3	4
$f(x)$	2	3	4	3	2

) The function f is continuous and differentiable on the closed interval [0,4]. The table above gives selected values of f on this interval. Which of the following statements must be true?

(A) The minimum value of f on [0,4] is 2.
(B) The maximum value of f on [0,4] is 4.
(C) $f(x)>0$ for $0<x<4$
(D) $f'(x)<0$ for $2<x<4$
(E) There exists c, with $0<c<4$, for which $f'(c)=0$.

16

) The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?

(A) $0.04\pi\ m^2/\text{sec}$
(B) $0.4\pi\ m^2/\text{sec}$
(C) $4\pi\ m^2/\text{sec}$
(D) $20\pi\ m^2/\text{sec}$
(E) $100\pi\ m^2/\text{sec}$

17

) The region bounded by the graph of $y=2x-x^2$ and the x-axis is the base of a solid. For this solid, each cross-section perpendicular to the x-axis is an equilateral triangle. What is the volume of this solid?

(A) 1.333
(B) 1.067
(C) 0.577
(D) 0.462
(E) 0.267

18) The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le t \le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

(A) $\int_{1.572}^{3.514} r(t)\,dt$

(B) $\int_{0}^{8} r(t)\,dt$

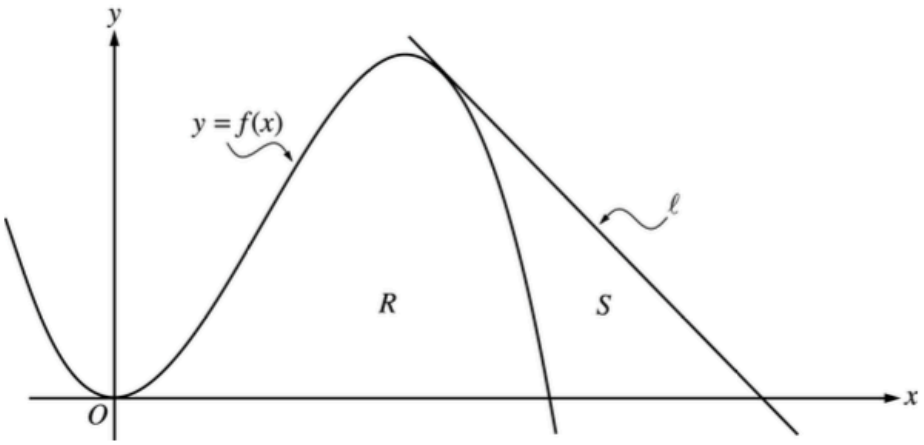
(C) $\int_{0}^{2.667} r(t)\,dt$

(D) $\int_{1.572}^{3.514} r'(t)\,dt$

(E) $\int_{0}^{2.667} r'(t)\,dt$

SECTION IIA: Calculator Free-Response (1 question—15 minutes)

19



Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the graph of f. Let R be the region bounded by the graph of f and the x-axis, and let S be the region bounded by the graph of f, the line ℓ, and the x-axis, as shown above.

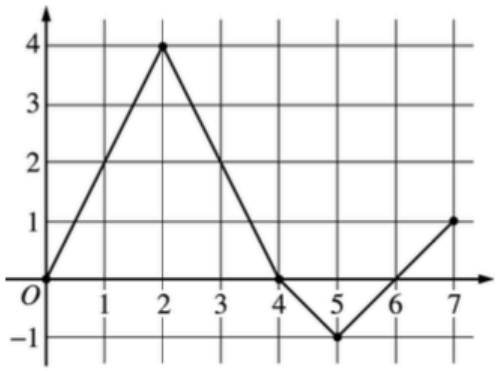
(a) Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.

(b) Find the area of S.

(c) Find the volume of the solid generated when R is revolved about the x-axis.

SECTION IIB: Non-Calculator Free-Response (2 questions—30 minutes)

20



Graph of f

Let f be a function defined on the closed interval $[0, 7]$. The graph of f, consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t)\,dt$.

(a) Find $g(3)$, $g'(3)$, and $g''(3)$.

(b) Find the average rate of change of g on the interval $0 \le x \le 3$.

(c) For how many values c, where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b) ? Explain your reasoning.

(d) Find the x-coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.

21

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	–220	150

Johanna jogs along a straight path. For $0 \le t \le 40$, Johanna's velocity is given by a differentiable function v. Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

(a) Use the data in the table to estimate the value of $v'(16)$.

(b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)|\,dt$ in the context of the problem.

Approximate the value of $\int_0^{40} |v(t)|\,dt$ using a right Riemann sum with the four subintervals indicated in the table.

(c) Bob is riding his bicycle along the same path. For $0 \le t \le 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute.

Find Bob's acceleration at time $t = 5$.

(d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \le t \le 10$.