APBC S.Hogan
Unit 5 Review

The topics on this review are similar to those found on the unit exam. Keep in mind that although the concepts will be the same, you may be required to apply them differently. In addition to this review, you should make sure that you are familiar with all material and formulae from this chapter.

Exam Format:

- Part A: 1 free-response (#11)—CALCULATOR REQUIRED
- Part B: 9 multiple-choice questions and 1 free-response (#10)—NON-CALCULATOR
- You will NOT be penalized for wrong answers on the multiple-choice, so you should answer EVERY question.

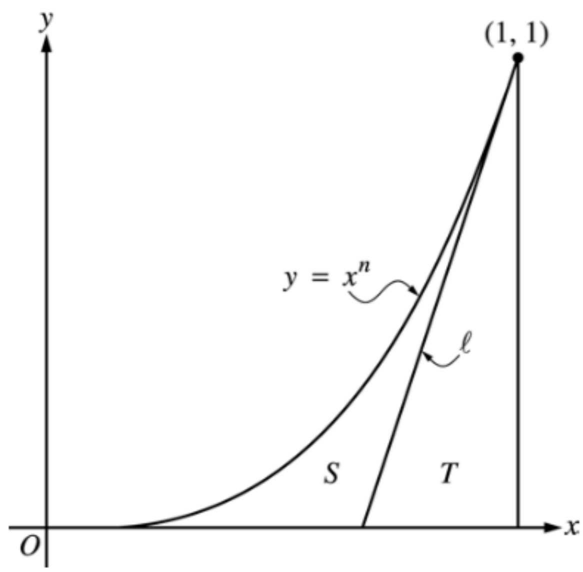
Time Restrictions:

You will have 48 minutes to complete this examination. This means that you are being required to move at AP pace (2 minutes per non-calculator multiple-choice and 15 minutes for free-response.) Specifically, you will have 15 minutes to do part A (the calculator section) and the remaining 33 minutes to do part B.

PLEASE COMPLETE THIS REVIEW SHEET IN ITS ENTIRETY BEFORE WE DO OUR IN-CLASS REVIEW. THAT IS THE BEST WAY TO PREPARE FOR THIS EXAM.

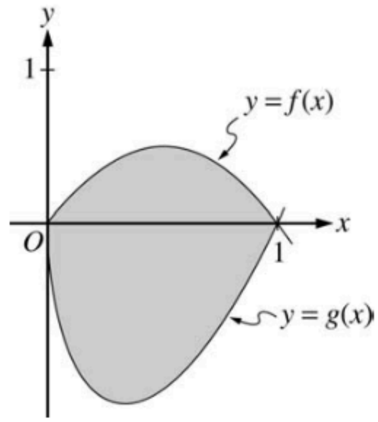
1	Which of the following integrals gives the length of the graph of $y = \tan x$ between $x = a$ and $x = b$, where $0 < a < b < \frac{\pi}{2}$? (A) $\int_a^b \sqrt{x^2 + \tan^2 x}\, dx$ (B) $\int_a^b \sqrt{x + \tan x}\, dx$ (C) $\int_a^b \sqrt{1 + \sec^2 x}\, dx$ (D) $\int_a^b \sqrt{1 + \tan^2 x}\, dx$ (E) $\int_a^b \sqrt{1 + \sec^4 x}\, dx$
2	The area of the region between the graph of $y = 4x^3 + 2$ and the x-axis from $x = 1$ to $x = 2$ is (A) 36 (B) 23 (C) 20 (D) 17 (E) 9

3	A region in the plane is bounded by the graph of $y = \frac{1}{x}$, the x-axis, the line $x = m$, and the line $x = 2m$, $m > 0$. The area of this region (A) is independent of m. (B) increases as m increases. (C) decreases as m increases. (D) decreases as m increases when $m < \frac{1}{2}$; increases as m increases when $m > \frac{1}{2}$. (E) increases as m increases when $m < \frac{1}{2}$; decreases as m increases when $m > \frac{1}{2}$.
4	What is the average (mean) value of $3t^3 - t^2$ over the interval $-1 \le t \le 2$? (A) $\frac{11}{4}$ (B) $\frac{7}{2}$ (C) 8 (D) $\frac{33}{4}$ (E) 16
5	A particle moves on the curve $y = \ln x$ so that the x-component has velocity $x'(t) = t + 1$ for $t \ge 0$. At time $t = 0$, the particle is at the point $(1,0)$. At time $t = 1$, the particle is at the point (A) $(2, \ln 2)$ (B) $(e^2, 2)$ (C) $\left(\frac{5}{2}, \ln\frac{5}{2}\right)$ (D) $(3, \ln 3)$ (E) $\left(\frac{3}{2}, \ln\frac{3}{2}\right)$
6	The acceleration α of a body moving in a straight line is given in terms of time t by $\alpha = 8 - 6t$. If the velocity of the body is 25 at $t = 1$ and if $s(t)$ is the distance of the body from the origin at time t, what is $s(4) - s(2)$? (A) 20 (B) 24 (C) 28 (D) 32 (E) 42
7	Suppose $g'(x) < 0$ for all $x \ge 0$ and $F(x) = \int_0^x t\,g'(t)\,dt$ for all $x \ge 0$. Which of the following statements is FALSE? (A) F takes on negative values. (B) F is continuous for all $x > 0$. (C) $F(x) = x\,g(x) - \int_0^x g(t)\,dt$ (D) $F'(x)$ exists for all $x > 0$. (E) F is an increasing function.

8	The base of a solid is the region in the first quadrant enclosed by the parabola $y=4x^2$, the line $x=1$, and the x-axis. Each plane section of the solid perpendicular to the x-axis is a square. The volume of the solid is (A) $\frac{4\pi}{3}$ (B) $\frac{16\pi}{5}$ (C) $\frac{4}{3}$ (D) $\frac{16}{5}$ (E) $\frac{64}{5}$
9	The region R in the first quadrant is enclosed by the lines $x=0$ and $y=5$ and the graph of $y=x^2+1$. The volume of the solid generated when R is revolved about the y-axis is (A) 6π (B) 8π (C) $\frac{34\pi}{3}$ (D) 16π (E) $\frac{544\pi}{15}$
10 FR	 Let ℓ be the line tangent to the graph of $y = x^n$ at the point $(1, 1)$, where $n > 1$, as shown above. (a) Find $\int_0^1 x^n\,dx$ in terms of n. (b) Let T be the triangular region bounded by ℓ, the x-axis, and the line $x = 1$. Show that the area of T is $\frac{1}{2n}$. (c) Let S be the region bounded by the graph of $y = x^n$, the line ℓ, and the x-axis. Express the area of S in terms of n and determine the value of n that maximizes the area of S.

11 FR

CALCULATOR REQUIRED (only on this problem)



Let f and g be the functions given by $f(x) = 2x(1 - x)$ and $g(x) = 3(x - 1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure above.

(a) Find the area of the shaded region enclosed by the graphs of f and g.

(b) Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.

(c) Let h be the function given by $h(x) = kx(1 - x)$ for $0 \le x \le 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.