AP BC Calculus (APBC) Unit 7 Review

The topics on this review are similar to those found on the unit exam. Keep in mind that although the concepts will be the same, you may be required to apply them differently. In addition to this review, you should make sure that you are familiar with all material and formulae from this chapter.

Exam Format:

- (1) 9 multiple-choice questions (NO CALCULATOR)—18 minutes
- (2) 2 free-response question (CALCULATOR)—30 minutes
- (3) You will NOT be penalized for wrong answers on the multiple-choice, so you should answer EVERY question.

<u>Time Restrictions:</u>

You will have 48 minutes to complete this examination. This means that you are being required to move at AP pace (2 minutes per non-calculator multiple-choice and 15 minutes for free-response.)

PLEASE COMPLETE THIS REVIEW SHEET IN ITS ENTIRETY BEFORE WE DO OUR IN-CLASS REVIEW. THAT IS THE BEST WAY TO PREPARE FOR THIS EXAM.

1	If $x = t^2 - 1$ and $y = 2e^t$, then $\frac{dy}{dx} =$
	(A) $\frac{e^{t}}{t}$ (B) $\frac{2e^{t}}{t}$ (C) $\frac{e^{ t }}{t^{2}}$ (D) $\frac{4e^{t}}{2t-1}$ (E) e^{t}
2	A particle moves in the <i>xy</i> -plane so that at any time <i>t</i> its coordinates are $x = t^2 - 1$ and $y = t^4 - 2t^3$. At $t = 1$, its acceleration vector is
	(A) $(0,-1)$ (B) $(0,12)$ (C) $(2,-2)$ (D) $(2,0)$ (E) $(2,8)$
3	The length of the curve determined by the equations $x = t^2$ and $y = t$ from $t = 0$ to $t = 4$ is
	(A) $\int_{0}^{4} \sqrt{4t+1} dt$ (B) $2\int_{0}^{4} \sqrt{t^{2}+1} dt$ (C) $\int_{0}^{4} \sqrt{2t^{2}+1} dt$ (D) $\int_{0}^{4} \sqrt{4t^{2}+1} dt$ (E) $2\pi \int_{0}^{4} \sqrt{4t^{2}+1} dt$
	(B) $2\int_{0}^{4}\sqrt{t^{2}+1} dt$
	(C) $\int_0^4 \sqrt{2t^2 + 1} dt$
	(D) $\int_0^4 \sqrt{4t^2 + 1} dt$
	(E) $2\pi \int_{0}^{4} \sqrt{4t^2 + 1} dt$

4	For what values of t does the curve given by the parametric equations $x = t^3 - t^2 - 1$ and $y = t^4 + 2t^2 - 8t$ have a vertical tangent?
	(A) 0 only(B) 1 only
	(C) 0 and $\frac{2}{3}$ only
	(D) 0, $\frac{2}{3}$, and 1
	(E) No value
5	A curve in the plane is defined parametrically by the equations $x = t^3 + t$ and $y = t^4 + 2t^2$. An equation of the line tangent to the curve at $t = 1$ is
	(A) $y = 2x$ (B) $y = 8x$ (C) $y = 2x - 1$
	(A) $y = 2x$ (B) $y = 8x$ (C) $y = 2x-1$ (D) $y = 4x-5$ (E) $y = 8x+13$
6	If $x = t^2 + 1$ and $y = t^3$, then $\frac{d^2 y}{dx^2} =$
	(A) $\frac{3}{4t}$ (B) $\frac{3}{2t}$ (C) $3t$ (D) $6t$ (E) $\frac{3}{2}$
7	
	The area of the region enclosed by the polar curve $r = 1 - \cos \theta$ is
	(A) $\frac{3}{4}\pi$ (B) π (C) $\frac{3}{2}\pi$ (D) 2π (E) 3π
8	The area of the closed region bounded by the polar graph of $r = \sqrt{3 + \cos \theta}$ is given by the integral
	(A) $\int_{0}^{2\pi} \sqrt{3 + \cos \theta} d\theta$ (B) $\int_{0}^{\pi} \sqrt{3 + \cos \theta} d\theta$ (C) $2 \int_{0}^{\pi/2} (3 + \cos \theta) d\theta$ (D) $\int_{0}^{\pi} (3 + \cos \theta) d\theta$ (E) $2 \int_{0}^{\pi/2} \sqrt{3 + \cos \theta} d\theta$
	(D) $\int_0^{\pi} (3 + \cos \theta) d\theta$ (E) $2 \int_0^{\pi/2} \sqrt{3 + \cos \theta} d\theta$
9	Which of the following is equal to the area of the region inside the polar curve $r = 2\cos\theta$ and outside the polar curve $r = \cos\theta$?
	(A) $3\int_{0}^{\frac{\pi}{2}}\cos^{2}\theta d\theta$ (B) $3\int_{0}^{\pi}\cos^{2}\theta d\theta$ (C) $\frac{3}{2}\int_{0}^{\frac{\pi}{2}}\cos^{2}\theta d\theta$ (D) $3\int_{0}^{\frac{\pi}{2}}\cos\theta d\theta$ (E) $3\int_{0}^{\pi}\cos\theta d\theta$

Free-Response Directions: Completely answer every portion of every question. You must show work where work was required to obtain the answer. Clearly box your answer for each portion of each question. All work must be in the space provided. No work on scrap paper will be counted.

10 An object moving along a curve in the xy-plane has position (x(t), y(t)) at time $t \ge 0$ with $\frac{dx}{dt} = 12t - 3t^2$ and $\frac{dy}{dt} = \ln(1 + (t - 4)^4)$. At time t = 0, the object is at position (-13, 5). At time t = 2, the object is at point P with x-coordinate 3. (a) Find the acceleration vector at time t = 2 and the speed at time t = 2. (b) Find the y-coordinate of P. (c) Write an equation for the line tangent to the curve at *P*. (d) For what value of t, if any, is the object at rest? Explain your reasoning. 11 2 $= \theta + \sin(2\theta)$ _ż 0 -3 The curve above is drawn in the xy-plane and is described by the equation in polar coordinates $r = \theta + \sin(2\theta)$ for $0 \le \theta \le \pi$, where r is measured in meters and θ is measured in radians. The derivative of r with respect to θ is given by $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$. (a) Find the area bounded by the curve and the x-axis. (b) Find the angle θ that corresponds to the point on the curve with x-coordinate -2. (c) For $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$, $\frac{dr}{d\theta}$ is negative. What does this fact say about r? What does this fact say about the curve? (d) Find the value of θ in the interval $0 \le \theta \le \frac{\pi}{2}$ that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answer.