CHAb - ch. 2 review sol'ns

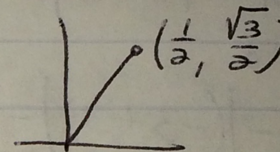
① $\lim_{x \to 1} (x-3) = 1-3 = -2$ (D)

② $\lim_{x \to -2} 1 = 1$ (D)

③ $\lim_{x \to 0} (x^3+2x^2-4x-6) = 0+0+0-6 = -6$ (D)

④ $\lim_{x \to -1} (2x^2-4x-3) = 2(-1)^2-4(-1)-3$
$= 2+4-3 = 6-3 = \boxed{3}$ (A)

⑤ $\lim_{x \to \pi/3} 2\csc x = 2\csc(\pi/3)$



$= 2\left(\frac{2}{\sqrt{3}}\right) = \frac{4}{\sqrt{3}}$

$= \frac{4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$ (C)

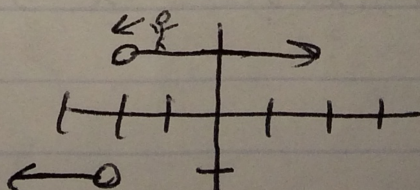
⑥ $\lim_{x \to -2} \frac{x+1}{x^2+x} = \frac{-2+1}{(-2)^2+(-2)} = \frac{-1}{4-2} = \boxed{\frac{-1}{2}}$ (D)

⑦ $\lim_{x \to -2^+} \frac{|x+2|}{x+2}$

NOTE: $\frac{|x+2|}{x+2} = \begin{cases} \frac{-(x+2)}{x+2} = -1, & x < -2 \\ \frac{x+2}{x+2} = 1, & x > -2 \end{cases}$

so for the right limit... use $\frac{x+2}{x+2} = 1$

$\boxed{\lim_{x \to -2^+} \frac{|x+2|}{x+2} = 1}$ OR on the graph



(D)

(8) $\lim_{x \to 1} f(x) = \boxed{DNE}$

$\lim_{x \to 1^-} f(x) = -1-6 = -7$

$\lim_{x \to 1^+} f(x) = -(1)^2 = -1$

$-1 \neq -7$

$f(x) = \begin{cases} -x-6, & x \le 1 \\ -x^2, & x > 1 \end{cases}$

(A)

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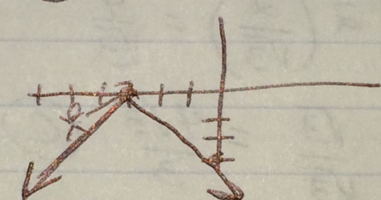
(9) $\lim_{x \to -3^-} -|x+3|$

absolute value is continuous fxn, so left lim = right lim = Double limit = fxn value so you can just plugin

$\lim_{x \to -3^-} -|x+3|$

$= -|(-3+3)| = \boxed{0}$ (B) OR graph it



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(10) $\lim_{x \to -3^-} f(x) = (-3)^2 + 8(-3) + 17 = 9 - 24 + 17$
$= -15 + 17 = \boxed{2}$

$f(x) = \begin{cases} x^2+8x+17, & x < -3 \\ -2x-4, & x \ge -3 \end{cases}$

(A)

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(11) $\lim_{x \to -3} \frac{x+3}{x^2+4x+3} = \frac{-3+3}{9-12+3} = \frac{0}{0}$ so you need to use algebra to simplify!!

$= \lim_{x \to -3} \frac{\cancel{x+3}}{(x+1)\cancel{(x+3)}} = \lim_{x \to -3} \frac{1}{x+1} = \frac{1}{-3+1} = \boxed{-\frac{1}{2}}$

(A)

(12) $\lim_{x\to 1} -\dfrac{x-1}{x^2+2x-3} = -\dfrac{1-1}{1+2-3} = -\dfrac{0}{0}$

so you need to use algebra to simplify

$= \lim_{x\to 1} -\dfrac{\cancel{(x-1)}}{(x+3)\cancel{(x-1)}}$

$= \lim_{x\to 1} -\dfrac{1}{(x+3)} = \dfrac{-1}{1+3} = \boxed{\dfrac{-1}{4}}$ (B)

(13) $\lim_{x\to 2} \dfrac{\sqrt{x+7}-3}{x-2} = \dfrac{\sqrt{9}-3}{2-2} = \dfrac{0}{0}$ ← so use Algebra to simplify

$= \lim_{x\to 2} \dfrac{(\sqrt{x+7}-3)}{(x-2)}\dfrac{(\sqrt{x+7}+3)}{(\sqrt{x+7}+3)} = \lim_{x\to 2} \dfrac{(x+7)-9}{(x-2)(\sqrt{x+7}+3)}$

$= \lim_{x\to 2} \dfrac{\cancel{(x-2)}}{\cancel{(x-2)}(\sqrt{x+7}+3)} = \lim_{x\to 2} \dfrac{1}{\sqrt{x+7}+3} = \dfrac{1}{\sqrt{2+7}+3}$

$= \dfrac{1}{\sqrt{9}+3} = \dfrac{1}{3+3} = \boxed{1/6}$ (D)

(14) $\lim_{x\to 0} \dfrac{x}{\frac{1}{-1+x}+1} = \dfrac{0}{-\frac{1}{1+0}+1} = \dfrac{0}{-1+1} = \dfrac{0}{0}$ so use Algebra to simplify

$= \lim_{x\to 0} \dfrac{x}{\frac{1}{(-1+x)} + \frac{1}{1}\frac{(-1+x)}{(-1+x)}} = \lim_{x\to 0} \dfrac{x}{\left(\frac{1+-1+x}{-1+x}\right)}$

$= \lim_{x\to 0} \dfrac{x}{\left(\frac{x}{-1+x}\right)} = \lim_{x\to 0} \cancel{x}\cdot\left(\dfrac{-1+x}{\cancel{x}}\right)$

$= \lim_{x\to 0} -1+x = -1+0 = \boxed{-1}$ (A)

CHAb-ch.2 review solns

15) $\lim_{x\to 0} \frac{3x}{\tan 4x} = \frac{3(0)}{\tan(0)} = \frac{3(0)}{0} = \frac{0}{0}$ so simplify w/ Algebra and trig

REMEMBER that $\lim_{x\to 0} \frac{\sin x}{x} = 1$

$$\lim_{x\to 0} \frac{3x}{\left(\frac{\sin 4x}{\cos 4x}\right)} = \lim_{x\to 0} 3x\left(\frac{\cos 4x}{\sin 4x}\right) = \lim_{x\to 0} \frac{3x\cos 4x}{\sin 4x}$$

$$= \lim_{x\to 0} 3\cos 4x \cdot \frac{x}{\sin 4x} = \lim_{x\to 0} 3\cos(4x) \cdot \lim_{x\to 0} \frac{x}{\sin 4x}$$

$$= \lim_{x\to 0} 3\cos 4x \cdot \frac{4}{4}\lim_{x\to 0} \frac{x}{\sin 4x}$$

WORK:

$$3\cos(0) = 3(1) = 3$$

$$\frac{4}{4}\lim_{x\to 0} \frac{x}{\sin 4x} = \frac{1}{4}\lim_{x\to 0} \frac{4x}{\sin 4x}$$

$$= \frac{1}{4}\lim_{x\to 0} \frac{1}{\left(\frac{\sin 4x}{4x}\right)}$$

$$= \frac{1}{4} \frac{\lim_{x\to 0} (1)}{\lim_{x\to 0} \frac{\sin 4x}{4x}}$$

$$= \frac{1}{4}\frac{(1)}{(1)} = \frac{1}{4}$$

$$3 \cdot \frac{1}{4} = \boxed{\frac{3}{4}}$$

B

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16) $\lim_{x\to 0} \frac{\sin x}{x} = 1$ (this is one of the limits that you should have memorized)

A

(17) $\lim\limits_{x\to 1} \dfrac{x^2}{x-1} = \dfrac{1}{1-1} = \dfrac{1}{0}$ ← so this will NOT simplify...

$x=1$ is a vertical asymptote ... IF

- the left lim is ∞ and the right lim is also ∞, then ANS is ∞.
- IF they (left and right) are both $-\infty$, then $-\infty$ is the answer.
- IF left and right limits go to DIFFERENT infinities, then the ANSWER is DNE

TO FIND out, make a table of x's (pick EASY ones) to the left AND right of $x=1$.

VA $F(x)=\dfrac{x^2}{x-1}$

TO THE LEFT (1) TO THE RIGHT

-1	0	2	3
$\frac{(-1)^2}{-1-1} = \frac{1}{-2}$	$\frac{0}{0-1} = 0$	$\frac{4}{2-1} = 4$	$\frac{9}{3-1} = 4.5$

From the left...going UP ∞

From the right, going DOWN $-\infty$

$\lim\limits_{x\to 1^-} f(x) = \infty$ $\lim\limits_{x\to 1^+} f(x) = -\infty$ (C)

$-\infty \neq \infty$ so $\lim\limits_{x\to 1} \dfrac{x^2}{x-1} = \boxed{DNE}$

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(18) $\lim\limits_{x\to 3} \dfrac{2}{x^2-9} = \dfrac{2}{9-9} = \dfrac{2}{0}$ SO IT WILL NOT simplify

make table just like in #17.

left right
$-\infty \neq \infty$

To the left (3) to the right

1	2	4	5
$\frac{2}{-8} = \frac{-1}{4}$	$\frac{2}{8-9} = -2$	$\frac{2}{16-9} = \frac{2}{7}$	$\frac{2}{25-9} = \frac{2}{16}$

VA

From left going down $-\infty$

∞ From right going up

$\boxed{DNE}$

(D)

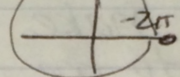
(19) $\lim\limits_{x \to 3^-} -\dfrac{x^2}{3x-9} = \dfrac{-9}{9-9} = \dfrac{-9}{0}$ So it won't simplify... it's a vertical asymptote

From the left...

1	2	3
$\frac{-1}{3-9} = 1/6$	$\frac{-4}{6-9} = 4/3$	VA

From the left going up $\nearrow \infty$

$\boxed{\infty}$ (D)

 $(1,0)$

(20) $\lim\limits_{x \to -\pi} \sec(2x) = \sec(-2\pi) = \dfrac{1}{\cos(-2\pi)} = \dfrac{1}{1}$

$= \boxed{1}$ (B)

plug in first $\dfrac{2-2}{4-6+2} = \dfrac{0}{0}$ so simplify

(21) $\lim\limits_{x \to 2^+} \dfrac{x-2}{x^2-3x+2} = \lim\limits_{x \to 2^+} \dfrac{x-2}{(x-2)(x-1)} = \lim\limits_{x \to 2^+} \dfrac{1}{x-1}$

so at $x=2$ there is a hole... which means that left limit = right limit at $x=2$ but the actual fxn is undefined...

(B)

so plug into simplified version

$\lim\limits_{x \to 2^+} \dfrac{x-2}{x^2-3x+2} = \lim\limits_{x \to 2^+} \dfrac{1}{x-1} = \dfrac{1}{2-1} = \boxed{1}$

(22) $\lim\limits_{x \to \frac{\pi}{4}^+} -2\tan(2x) = -2\tan(\pi/2) = -2\dfrac{\sin(\pi/2)}{\cos \pi/2}$

$= \dfrac{-2}{0}$ ← so it won't simplify... it's an asymptote to the right

from the right

$\pi/4$ ~~$\pi/2$~~	$2\pi/3$
$-2\tan\pi = 0$	$-2\tan(4\pi/3) = -2\left(\frac{\sqrt{3}}{1}\right)$

∞ going up from the right

$\boxed{\infty}$ (A)

(23) $\lim\limits_{x \to -\infty} \dfrac{x}{x^2+2x+2}$

LIMITS AS $x \to \pm\infty$ are horizontal asymptotes that deal with really large values of x

TRICK: For really large (infinite) values of x, adding a finite number is insignificant (for example $\infty + 3 = \infty$)
so, we're going to use logic to simplify after we plug in ∞

$$\lim_{x \to -\infty} \frac{x}{(x^2+2x+2)} = \frac{-\infty}{(-\infty)^2+2(-\infty)+2}$$

FINITE # IS INSIGNIFICANT

behaves like:

$$= \frac{-\infty}{\infty^2 - 2\infty} = \frac{\infty(-1)}{\infty(\infty-2)} = \frac{-1}{\infty - 2}$$

FINITE # IS insignificant

behaves like:

$$= \frac{-1}{\infty} = \boxed{0}$$

(when you divide a finite # by an infinite #, you get 0)

OR: OFFICIAL WAY... Find largest power of x... call it x^n. multiply every single term by $\frac{1}{x^n}$ (meaning you multiplied the entire expression by $\frac{1/x^n}{1/x^n}$)... simplify and use the rule that $\lim\limits_{x \to \pm\infty} \dfrac{c}{x^n} = 0$

$$\lim_{x \to -\infty} \frac{x}{x^2+2x+2} \cdot \frac{\left(\frac{1}{x^2}\right)}{\left(\frac{1}{x^2}\right)}$$

$$= \lim_{x \to -\infty} \frac{\frac{x}{x^2}}{\frac{x^2}{x^2} + \frac{2x}{x^2} + \frac{2}{x^2}} = \lim_{x \to -\infty} \frac{\frac{1}{x}}{1 + \frac{2}{x} + \frac{2}{x^2}} = \frac{0}{1+0+0} = \frac{0}{1}$$

$$= \boxed{0}$$

(C)

24 (like # 23)

$$\lim_{x\to\infty} \frac{25x}{x^2+25}$$

"Logic" method:

$$\frac{25\infty}{\infty^2+25} \text{ behaves like } \frac{25\infty}{\infty^2} = \frac{25}{\infty} = \boxed{0}$$

Algebraic method:

$$\lim_{x\to\infty} \frac{25x\,(1/x^2)}{x^2+25\,(1/x^2)} = \lim_{x\to\infty} \frac{\frac{25x}{x^2}}{\frac{x^2}{x^2}+\frac{25}{x^2}}$$

$$= \lim_{x\to\infty} \frac{25/x}{1+25/x^2} = \frac{\lim_{x\to\infty} 25/x}{\lim_{x\to\infty} 1 + \lim_{x\to\infty} 25/x^2} = \frac{0}{1+0} = 0$$

$$= \boxed{0}$$ A

(Also like #23)

25 $$\lim_{x\to\infty} \frac{2x-3}{\sqrt{2x^2+2}}$$ B

"logic" method:

$$\frac{2\infty-3 \leftarrow \text{FINITE}}{\sqrt{2\infty^2+2}\ \nwarrow \text{FINITE}} \text{ behaves like } \frac{2\infty}{\sqrt{2\infty^2}} = \frac{2\cancel{\infty}}{\cancel{\infty}\sqrt{2}} = \frac{2}{\sqrt{2}} = \frac{\cancel{2}\sqrt{2}}{\cancel{2}} = \boxed{\sqrt{2}}$$

Algebraic method:

$$\lim_{x\to+\infty} \frac{2x-3 \cdot 1/x}{\sqrt{2x^2+2} \cdot 1/x} = \lim_{x\to\infty} \frac{\frac{2x}{x}-\frac{3}{x}}{\sqrt{\frac{2x^2}{x^2}+\frac{2}{x^2}}}$$

which is $\sqrt{\frac{1}{x^2}}$

$$= \lim_{x\to\infty} \frac{2-3/x}{\sqrt{2+2/x^2}} = \frac{2-0}{\sqrt{2+0}} = \frac{2}{\sqrt{2}} = \frac{\cancel{2}\sqrt{2}}{\cancel{2}} = \boxed{\sqrt{2}}$$

(26) (like 25)

D

$$\lim_{x\to-\infty} \frac{x}{\sqrt{2x^2+4}}$$

"logic" method:

$$\frac{-\infty}{\sqrt{2\infty^2+4}}$$ behaves like $$\frac{-\infty}{\sqrt{2\infty^2}} = \frac{-\cancel{\infty}}{\cancel{\infty}\sqrt{2}} = \frac{-1}{\sqrt{2}} = \boxed{\frac{-\sqrt{2}}{2}}$$

↑ FINITE

Algebraic method:

$$\lim_{x\to-\infty} \frac{x}{\sqrt{2x^2+4}} \cdot \frac{\frac{1}{x}}{-\sqrt{\frac{1}{x^2}}} = \lim_{x\to-\infty} \frac{-\frac{x}{x}}{\sqrt{\frac{2x^2}{x^2}+\frac{4}{x^2}}} =$$

↑ which are the same because x is $-$

D

$$= \lim_{x\to-\infty} \frac{-1}{\sqrt{2+4/x^2}} = \frac{-1}{\sqrt{2+0}} = \frac{-1}{\sqrt{2}} = \boxed{\frac{-\sqrt{2}}{2}}$$

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(27) $f(x) = \begin{cases} x^2+1, & x<1 \\ -\frac{x}{2}+\frac{3}{2}, & x\geq 1 \end{cases}$

x^2+1 is always continuous, $-\frac{x}{2}+\frac{3}{2}$ is also always continuous, so we just have to check the "break point"

$\lim_{x\to1^-} f(x) = 1^2+1 = 2$ $\lim_{x\to1^+} f(x) = -\frac{1}{2}+\frac{3}{2} = 1$

$2 \neq 1$ so $f(x)$ is Discontinuous at $x=1$

D $(-\infty,1) \cup [1,\infty)$

(28) $f(x) = \dfrac{x-4}{(x^2+3x)} = \dfrac{x-4}{(x+3)(x)}$

VA at $x=-3$ VA at $x=0$

$\boxed{\text{continuous on } (-\infty,-3) \cup (-3,0) \cup (0,\infty)}$

(D)

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(29) $f(x) = \begin{cases} x^2+2x+2, & x \le 0 \\ -x, & x > 0 \end{cases}$

x^2+2x+2 is continuous on ITS DOMAIN
$-x$ is also continuous on ITS DOMAIN
so we just need to check the "break point"

$\lim_{x \to 0^-} f(x) = 0+0+2 = 2$

$\lim_{x \to 0^+} f(x) = -0 = 0$

$2 \neq 0$ $\boxed{\text{Jump Discontinuity at } x=0}$

(B)

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(30) $f(x) = 2\tan(2x)$ $[-\pi, \pi]$

$= \dfrac{2\sin 2x}{\cos 2x}$ ← VA when $\cos 2x = 0$

$2x = \underbrace{\dfrac{-3\pi}{2}, \dfrac{-\pi}{2}}_{\text{once}}, \underbrace{\dfrac{\pi}{2}, \dfrac{3\pi}{2}}_{\text{twice}}$

go twice around interval.

$\boxed{x = \dfrac{-3\pi}{4}, \dfrac{-\pi}{4}, \dfrac{\pi}{4}, \dfrac{3\pi}{4}}$

(D)

vertical Asymptotes are also called essential or non-removable DISCONTINUITIES

31) $f(x) = \cos\left(\frac{1}{x-\pi}\right)$

cosine is a continuous fxn but inside the cosine, there's an $x-\pi$ in the denominator.

So, $x - \pi \neq 0$
$x \neq \pi$

π is the only problem spot...but it IS a problem...

So ans must be (C) $x = \pi$ (oscillating discontinuity)

this is because as $x \to \pi$, $\frac{1}{x-\pi} \to \infty$ and cosine oscillates forever between -1 and 1.

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32) $f(x) = \frac{x}{x^2-2x-3} = \frac{x}{(x-3)(x+1)}$

VA: $x-3=0$, $x=3$
VA: $x+1=0$, $x=-1$

So vertical asymptotes (a.k.a. essential or non-removable discontinuities) at $x = 3, -1$ (A)

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33) $\lim_{x\to-2} \frac{x}{\frac{1}{2+x} - \frac{1}{2}} = \frac{-2}{\frac{1}{2-2} - \frac{1}{2}} = \frac{-2}{\frac{1}{0} - \frac{1}{2}}$ problem...

even though it's not $0/0$, we should simplify by getting a common denominator

$= \lim_{x\to-2} \frac{x}{\frac{2}{2}\left(\frac{1}{2+x}\right) - \frac{1}{2}\left(\frac{2+x}{2+x}\right)} = \lim_{x\to-2} \frac{x}{\frac{2-2-x}{2(2+x)}}$

$= \lim_{x\to-2} \frac{x}{\left(\frac{-x}{2(2+x)}\right)} = \lim_{x\to-2} \cancel{x} \cdot \frac{2(2+x)}{-\cancel{x}} = \lim_{x\to-2} \frac{2(2-2)}{-1}$

$= \frac{2(0)}{-1} = \boxed{0}$ (D)

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(34) $\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1} = \frac{\sqrt{1}-1}{1-1} = \frac{0}{0}$ so simplify

You can rationalize:

$$\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}+1}\right) = \lim_{x\to 1} \frac{\cancel{x-1}}{\cancel{(x-1)}(\sqrt{x}+1)}$$

$$= \lim_{x\to 1} \frac{1}{\sqrt{x}+1} = \frac{1}{\sqrt{1}+1} = \boxed{\frac{1}{2}}$$ A

OR FACTOR (if you spot that $x-1 = \sqrt{x}^2 - 1^2$)

$$\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1} = \lim_{x\to 1} \frac{\cancel{\sqrt{x}-1}}{\cancel{(\sqrt{x}-1)}(\sqrt{x}+1)} = \frac{1}{\sqrt{1}+1} = \boxed{\frac{1}{2}}$$

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(35) $\lim_{x\to 3} \frac{\sqrt{x+6}-3}{x-3} = \frac{\sqrt{9}-3}{3-3} = \frac{0}{0}$ so simplify...

$$= \lim_{x\to 3} \frac{\sqrt{x+6}-3}{(x-3)}\left(\frac{\sqrt{x+6}+3}{\sqrt{x+6}+3}\right) = \lim_{x\to 3} \frac{(x+6)-9}{(x-3)(\sqrt{x+6}+3)}$$

$$= \lim_{x\to 3} \frac{\cancel{x-3}}{\cancel{(x-3)}(\sqrt{x+6}+3)} = \frac{1}{\sqrt{9}+3} = \boxed{\frac{1}{6}}$$ A
