Calculus Honors (CHAB) S.Hogan
Ch.4 Review

This review is identical to the exam in format. Only the actual values in the questions will vary.

CALCULATORS ARE NOT PERMITTED.

Multiple-Choice—(60 points)—15 questions worth 4 points each. No partial credit will be awarded. Only the answer selected will matter. See MC packet.

Free-Response—(20 points)—Two questions worth 10 points each. Partial credit will be given for correct work.

Free-Response: (Total 40 points…exact points are listed in *italics* in each problem.)
You must show a reasonable amount of work that leads to your answer. Where it is impossible to show work, explain the mental leaps that you made to draw your conclusion. (10 points each)

16) A conical paper cup is 20 cm tall with a radius of 10 cm. The bottom of the cup is punctured so that the water leaks out at a rate of $\frac{9\pi}{4}$ cm³/sec. At what rate is the water level changing when the water level is 6 cm?

17) Which points on the graph of $y = 7 - x^2$ are closest to the point $(0, 3)$?

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For each problem, find the equation of the line tangent to the function at the given point. Your answer should be in slope-intercept form.

1) $y = -(x+1)^{\frac{1}{3}}$ at $(-2, 1)$

A) $y = -\frac{\sqrt[3]{3}}{9}x + \frac{5\sqrt[3]{3}}{9}$

B) $y = -\frac{\sqrt[3]{2}}{6}x - \frac{5\sqrt[3]{2}}{6}$

C) $y = \frac{1}{5}x - 2$

D) $y = -\frac{1}{3}x + \frac{1}{3}$

For each problem, find the equation of the line normal to the function at the given point. If the normal line is a vertical line, indicate so. Otherwise, your answer should be in slope-intecept form.

2) $y = -x^3 + 3x^2 - 6$ at $(2, -2)$

A) Normal line is vertical line at $x = 2$

B) $y = \frac{1}{9}x - \frac{17}{9}$

C) $y = \frac{1}{9}x - \frac{19}{3}$

D) $y = \frac{1}{24}x + \frac{169}{12}$

For each problem, find the derivative of the function at the given value.

3) $f(x) = \frac{2}{x-1}$ at $x = -1$

A) $f'(-1) = -\frac{2}{9}$

B) $f'(-1) = -2$

C) $f'(-1) = -\frac{2}{25}$

D) $f'(-1) = -\frac{1}{2}$

For each problem, find the slope of the function at the given value.

4) $f(x) = -x^2 + 4x$ at $x = 1$

A) 12 B) -4

C) 2 D) 10

For each problem, find the points where the tangent line to the function is horizontal.

5) $y = -\frac{16x}{x^2 + 16}$

A) No horizontal tangent line exits.

B) $\left(6, -\frac{24}{13}\right)$

C) $\left(3, -\frac{48}{25}\right), (4, -2)$

D) $(-4, 2), (4, -2)$

For each problem, find the slope of the function at the given value.

6) $y = \cot(x)$ at $x = \frac{\pi}{3}$

A) $-\frac{4}{3}$ B) 1

C) -2 D) -1

For each problem, find the equation of the line tangent to the function at the given point. Your answer should be in slope-intercept form.

7) $f(x) = -\csc(x)$ at $\left(-\frac{\pi}{2}, 1\right)$

A) $y = 1$

B) $y = \sqrt{2} \cdot x + \frac{4\sqrt{2} - 7\pi\sqrt{2}}{4}$

C) $y = -1$

D) $y = -\sqrt{2} \cdot x + \frac{4\sqrt{2} + 5\pi\sqrt{2}}{4}$

For each problem, find the equation of the line normal to the function at the given point. If the normal line is a vertical line, indicate so. Otherwise, your answer should be in slope-intecept form.

8) $y = -\cot(x)$ at $\left(-\frac{\pi}{3}, \frac{\sqrt{3}}{3}\right)$

A) $y = -x + \frac{\pi}{2}$

B) $y = -\frac{1}{2}x + \frac{-8 + 5\pi}{8}$

C) $y = -\frac{3}{4}x + \frac{4\sqrt{3} - 3\pi}{12}$

D) $y = -\frac{1}{2}x + \frac{8 + 7\pi}{8}$

For each problem, find the values of c that satisfy the Mean Value Theorem.

9) $y = x^2 - 4x + 5; \quad [1, 4]$

A) $\{3\}$ B) $\left\{\frac{7}{2}\right\}$

C) $\left\{\frac{5}{2}\right\}$ D) $\{2\}$

For each problem, determine if the Mean Value Theorem can be applied. If it can, find all values of c that satisfy the theorem. If it cannot, explain why not.

10) $f(x) = \frac{x^2 - 4}{4x}; \quad [-3, -1]$

A) $\{-\sqrt{3}\}$
B) The function is not continuous on $[-3, -1]$
C) The function is not differentiable on $(-3, -1)$
D) $\{-\sqrt{5}\}$

For each problem, find the open intervals where the function is increasing and decreasing.

11) $f(x) = x^3 - 4x^2 + 5$

A) Increasing: $\left(\frac{1}{3}, \frac{8}{9}\right)$ Decreasing: $\left(-\infty, \frac{1}{3}\right), \left(\frac{8}{9}, \infty\right)$

B) Increasing: $(-\infty, 0), \left(\frac{8}{3}, \infty\right)$ Decreasing: $\left(0, \frac{8}{3}\right)$

C) Increasing: $\left(0, \frac{8}{3}\right)$ Decreasing: $(-\infty, 0), \left(\frac{8}{3}, \infty\right)$

D) Increasing: $(-\infty, 4), \left(\frac{32}{3}, \infty\right)$ Decreasing: $\left(4, \frac{32}{3}\right)$

For each problem, find the open intervals where the function is concave up and concave down.

12) $y = -2\cot(x); \quad [-\pi, \pi]$

A) Concave up: $\left(-\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, \pi\right)$ Concave down: $\left(-\pi, -\frac{\pi}{2}\right), \left(0, \frac{\pi}{2}\right)$

B) Concave up: $(0, \pi), \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ Concave down: $(-\pi, 0), \left(-\pi, -\frac{\pi}{2}\right), \left(\frac{\pi}{2}, \pi\right)$

C) Concave up: $\left(-\pi, -\frac{\pi}{2}\right), \left(\frac{\pi}{2}, \pi\right)$ Concave down: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

D) Concave up: $\left(-\pi, -\frac{\pi}{2}\right), \left(0, \frac{\pi}{2}\right)$ Concave down: $\left(-\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, \pi\right)$

For each problem, find the x-coordinates of all points of inflection and find the open intervals where the function is concave up and concave down.

13) $y = -x^2 - 8x - 13$

A) No inflection points exist.
Concave up: $(-\infty, \infty)$ Concave down: No intervals exist.

B) Inflection point at: $x = 2$
Concave up: $(-\infty, 2)$ Concave down: $(2, \infty)$

C) Inflection point at: $x = 3$
Concave up: $(-\infty, 3)$ Concave down: $(3, \infty)$

D) No inflection points exist.
Concave up: No intervals exist. Concave down: $(-\infty, \infty)$

For each problem, find all points of absolute minima and maxima on the given interval.

14) $y = (-2x + 8)^{\frac{1}{2}}$; $[-2, 2]$

A) Absolute minimum: $(4, 0)$
No absolute maxima.

B) Absolute minimum: $(-2, 2\sqrt{3})$
Absolute maximum: $(4, 0)$

C) Absolute minimum: $(2, 2)$
Absolute maximum: $(-2, 2\sqrt{3})$

D) No absolute minima.
No absolute maxima.

Solve each related rate problem.

15) Water slowly evaporates from a circular shaped puddle. The area of the puddle decreases at a rate of 81π in²/hr. Assuming the puddle retains its circular shape, at what rate is the radius of the puddle changing when the radius is 13 in?

A) A = area of circle r = radius t = time

Equation: $A = \pi r^2$ Given rate: $\frac{dA}{dt} = -81\pi$ Find: $\left.\frac{dr}{dt}\right|_{r=13}$

$$\left.\frac{dr}{dt}\right|_{r=13} = \frac{1}{2\pi r} \cdot \frac{dA}{dt} = -\frac{81}{26} \text{ in/hr}$$

B) A = area of circle r = radius t = time

Equation: $A = \pi r^2$ Given rate: $\frac{dA}{dt} = -81\pi$ Find: $\left.\frac{dr}{dt}\right|_{r=13}$

$$\left.\frac{dr}{dt}\right|_{r=13} = \frac{1}{2\pi r} \cdot \frac{dA}{dt} = -\frac{73}{25} \text{ in/hr}$$

C) A = area of circle r = radius t = time

Equation: $A = \pi r^2$ Given rate: $\frac{dA}{dt} = -81\pi$ Find: $\left.\frac{dr}{dt}\right|_{r=13}$

$$\left.\frac{dr}{dt}\right|_{r=13} = \frac{1}{2\pi r} \cdot \frac{dA}{dt} = -\frac{43}{13} \text{ in/hr}$$

D) A = area of circle r = radius t = time

Equation: $A = \pi r^2$ Given rate: $\frac{dA}{dt} = -81\pi$ Find: $\left.\frac{dr}{dt}\right|_{r=13}$

$$\left.\frac{dr}{dt}\right|_{r=13} = \frac{1}{2\pi r} \cdot \frac{dA}{dt} = -\frac{85}{27} \text{ in/hr}$$

16) A conical paper cup is 20 cm tall with a radius of 10 cm. The bottom of the cup is punctured so that the water leaks out at a rate of $\frac{9\pi}{4}$ cm³/sec. At what rate is the water level changing when the water level is 6 cm?

Solve each optimization problem.

17) Which points on the graph of $y = 7 - x^2$ are closest to the point $(0, 3)$?

Answers to Ch.4 Review (ID: 1)

1) D
2) A
3) D
4) C
5) D
6) A
7) A
8) C
9) C
10) A
11) B
12) A
13) D
14) C
15) A
16) $-\frac{1}{4}$ cm/sec
17) $\left(-\frac{\sqrt{14}}{2}, \frac{7}{2}\right), \left(\frac{\sqrt{14}}{2}, \frac{7}{2}\right)$