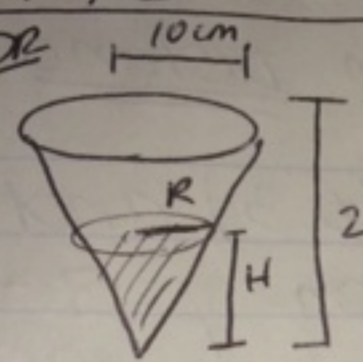
CHAB - ch. 4 review

(16) DR



$\frac{dV}{dT} = -\frac{9\pi}{4} cm^3/sec$

WANT: $\frac{dH}{dT}$ when $H = 6cm$

E: $V = \frac{1}{3}\pi r^2 h$

need Auxillary EQ to get rid of r.

$\frac{r}{h} = \frac{10}{20}$

$r = \frac{1}{2}h = \frac{h}{2}$

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$V = \frac{1}{3}\pi \frac{h^2}{4} \cdot h = \frac{\pi}{12}h^3$

E: $V = \frac{\pi}{12}h^3$

D: $\frac{dV}{dt} = \frac{3\pi}{12}h^2\frac{dh}{dT} = \frac{\pi}{4}h^2\frac{dh}{dT}$

S: $-\frac{9\pi}{4} = \frac{\pi}{4}(6)^2\frac{dh}{dT}$

$-\frac{9\pi}{4} = \frac{36\pi}{4}\frac{dh}{dT}$

$-\frac{9\cancel{\pi}}{4} \cdot \frac{4}{36\cancel{\pi}} = \frac{dh}{dT}$

$-\frac{9}{36} = \boxed{-\frac{1}{4}\frac{cm}{sec} = \frac{dh}{dT}}$

(17) $y = 7 - x^2$ closest to POINT $(0,3)$

"closest" means minimum Distance

distance, d, between $(0,3)$ and mystery point (x,y) on $y = 7 - x^2$ is

$$d = \sqrt{(x-0)^2 + (y-3)^2}$$

$$d = \sqrt{x^2 + (y-3)^2}$$

need to replace x or y...

$y = 7 - x^2$

$x^2 = 7 - y$

$$d = \sqrt{(7-y) + (y-3)(y-3)}$$

$$d = \sqrt{7 - y + y^2 - 6y + 9} = \sqrt{y^2 - 7y + 16}$$

TO FIND EXTREMA OF d, you take

① the derivative...

$$d' = \tfrac{1}{2}(y^2 - 7y + 16)^{-1/2}(2y - 7)$$

$$d' = \frac{2y - 7}{2\sqrt{y^2 - 7y + 16}}$$

② FIND CRITICAL POINTS

Ⓐ $d' = 0$

$2y - 7 = 0$

$y = 7/2$

Ⓑ d' DNE

$2\sqrt{y^2 - 7y + 16} = 0$

$2d = 0$

NOT logical...

$d \neq 0$

b/c $(0,3)$ IS NOT ON $y = 7 - x^2$

Ⓒ NO endpts

③ d' sign chart

	$-\infty$	$7/2$	∞
$2x-7$	$-$		$+$
$2\sqrt{y^2-7y+16}$	$+$		$+$
d'	$\ominus$		$\oplus$
d	$\searrow$	MIN	$\nearrow$

MINIMUM distance is when

$y = 7/2$

$x^2 = 7 - y$

$x^2 = 7 - \frac{7}{2}$

$x^2 = \frac{14}{2} - \frac{7}{2} = \frac{7}{2}$

$x = \pm\sqrt{7/2}$

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17 (cont.)

minimum distance occurs at the points

$$\boxed{\left(\sqrt{\tfrac{7}{2}}, \tfrac{7}{2}\right) \text{ AND } \left(-\sqrt{\tfrac{7}{2}}, \tfrac{7}{2}\right)}$$

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(1) $y = -(x+1)^{1/3}$ at $(-2,1)$

$y' = -\frac{1}{3}(x+1)^{-2/3}(1)$

$y' = \frac{-1}{3(x+1)^{2/3}}$ $\quad m_T = \frac{-1}{3(-1)^{2/3}} = \frac{-1}{3}$

$(-1)^\wedge(2/3) =$
$((-1)^2)^{1/3} = (1)^{1/3} = 1$

$m_T = \frac{-1}{3}$ (so it has to be answer D)

$y - 1 = \frac{-1}{3}(x+2)$ $\quad$ D

$y - 1 = -\frac{1}{3}x - \frac{2}{3}$

$y = \frac{-1}{3}x - \frac{2}{3} + 1 = \boxed{\frac{-1}{3}x + \frac{1}{3} = y}$

(2) $y = -x^3 + 3x^2 - 6$ at $(2,-2)$

$y' = -3x^2 + 6x$ $\quad m_T = -3(4) + 6(2)$

$m_T = -12 + 12$

$m_T = 0 \leftarrow$ so tan line is horizontal

m_n is undefined
(so normal line is a
vertical line through $(\underset{x}{2}, \underset{y}{-2})$...)

ANS: $\boxed{x = 2}$ $\quad$ A

(3) $f(x) = \frac{2}{x-1}$ @ $x = -1$
You can either use quotient rule
with $H = 2$, $L = x - 1$ or use...

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(3) (cont.) .. the chain rule by rewriting $(x-1)^{-1}$...

I pick quotient

$$f(x) = \frac{2}{x-1} \begin{matrix} \leftarrow H \\ \leftarrow L \end{matrix}$$

$$f'(x) = \frac{(x-1)(0) - (2)(1)}{(x-1)^2} = \frac{-2}{(x-1)^2}$$

$$f'(-1) = \frac{-2}{(-1-1)^2} = \frac{-2}{(4)} = \frac{-1}{2} \quad D$$

You can check this in a calculator by putting $Y_1 = 2/(x-1)$

$\boxed{\text{math}}\ \boxed{8} \rightarrow$ NDeriv $(Y_1, x, -1) = -\frac{1}{2}$

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(4) $f(x) = -x^2 + 4x \quad @\ x = 1$

$f'(x) = -2x + 4$ (C)

$m_T = f'(1) = -2(1) + 4 = \boxed{2}$

check with NDeriv $(-x^2 + 4x, x, 1) = 2$ ✓

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(5) $$y = -\frac{16x}{x^2+16} = \frac{-16x}{x^2+16} \begin{matrix} \leftarrow H \\ \leftarrow L \end{matrix}$$

$$y' = \frac{(x^2+16)(-16) - (-16x)(2x)}{(x^2+16)^2}$$

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⑤ (CONT.)

$$y' = \frac{(x^2+16)(-16) - (-16x)(2x)}{(x^2+16)^2}$$

$$y' = \frac{-16x^2 - 256 + 32x^2}{(x^2+16)^2} = \frac{16x^2-256}{(x^2+16)^2}$$

$$y' = \frac{16(x^2-16)}{(x^2+16)^2} = \frac{16(x+4)(x-4)}{(x^2+16)^2}$$

$$y' = 0 = 16(x+4)(x-4)$$

$16=0$ lie $\quad$ $x+4=0$ $\quad$ $x-4=0$

$\boxed{x=-4 \quad x=4}$ ← so has to be D...

$\underline{x=-4}$

$$y = \frac{-16(-4)}{(16+16)} = \frac{-\cancel{16}(-4)}{2(\cancel{16})} = 2 \qquad \boxed{(-4,2)}$$

$\underline{x=4}$

$$y = \frac{-16(4)}{16+16} = \frac{-\cancel{16}(4)}{2(\cancel{16})} = -2 \qquad \boxed{(4,2)}$$

Ⓓ

~~⑥ ... f(x) @ x = π/3~~

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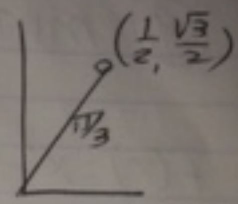
(6) $y = \cot x$ @ $\pi/3 = x$

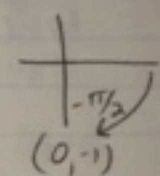


$y' = -\csc^2 x$

$y'(\frac{\pi}{3}) = -(\csc \frac{\pi}{3})^2 = -(\frac{2}{\sqrt{3}})^2 = \boxed{-\frac{4}{3}}$

(A)

(7) $f(x) = -\csc x$ at $(-\frac{\pi}{2}, 1)$



$f'(x) = \csc x \cot x$

$m_T = f'(-\frac{\pi}{2}) = \csc(-\frac{\pi}{2}) \cot(-\frac{\pi}{2})$

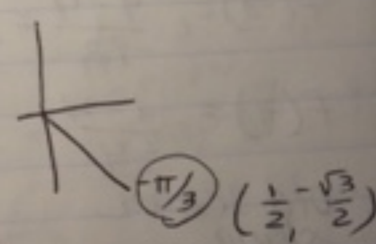
$m_T = \frac{1}{-1} \cdot \frac{0}{-1} = 0$ ← tangent line is horizontal

so ANS: ← through $(-\frac{\pi}{2}, 1)$ x, y

$\boxed{y = 1}$ (A)

(8) $y = -\cot x$ at $(-\pi/3, \sqrt{3}/2)$

NORMAL



$y' = +\csc^2 x$

$m_T = (-\frac{2}{\sqrt{3}})^2 = \frac{4}{3}$

$m_n = -\frac{3}{4}$ ← so it has to be C

$-\frac{3\pi}{12} + \frac{4\sqrt{3}}{12}$

$y - \frac{\sqrt{3}}{2} = -\frac{3}{4}(x + \frac{\pi}{3})$

$\boxed{y = -\frac{3}{4}x - \frac{\pi}{4} + \frac{\sqrt{3}}{2}}$ (C)

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(9) MVT: $\frac{f(b)-f(a)}{b-a} = f'(c)$

$f(x) = x^2 - 4x + 5 \quad [1, 4]$

$f(1) = 1 - 4 + 5 = 2 \rightarrow (1, 2)$

$f(4) = 16 - 16 + 5 = 5 \quad (4, 5)$

old school slope

$\frac{5-2}{4-1} = \frac{3}{3} = 1$

Derivative

$f'(x) = 2x - 4$

$f'(c) = 2c - 4$

$\rightarrow 1 = 2c - 4$

$5 = 2c$

$\boxed{5/2 = c}$

(C)

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(10) $f(x) = \frac{x^2 - 4}{4x} \quad [-3, -1]$

← only discontinuity is at $4x = 0 \rightarrow x = 0$ .. which is NOT in $[-3, -1]$

So MVT applies
(meaning it's NOT B or C)

$f(-3) = \frac{9-4}{-12} = \frac{5}{-12} \longrightarrow (-3, 5/{-12})$

$f(-1) = \frac{1-4}{-4} = \frac{-3}{-4} = \frac{3}{4} \rightarrow (-1, 3/4)$

old school
slope ..

$\frac{\frac{3}{4} + \frac{5}{12}}{-1+3} = \frac{\frac{9}{12} + \frac{5}{12}}{2} = \frac{14}{12} \cdot \frac{1}{2}$

$= \frac{7}{12}$

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(10) (cont.)

old school slope $= \frac{7}{12}$

Derivative

$$f(x) = \frac{x^2-4}{4x} \begin{matrix} \leftarrow H \\ \leftarrow L \end{matrix}$$

$$f'(x) = \frac{4x(2x) - (x^2-4)(4)}{(4x)^2}$$

$$= \frac{8x^2 - 4x^2 + 16}{16x^2} = \frac{\cancel{4}(x^2+4)}{\cancel{4}(4x^2)} = \frac{4x^2+16}{16x^2}$$

$$f'(c) = \frac{c^2+4}{4c^2} \leftarrow \text{instant slope AT } c$$

MVT:

$$\frac{7}{12} = \frac{c^2+4}{4c^2}$$

$$28c^2 = 12c^2 + 48$$
$$-12c^2 \quad -12c^2$$

$$16c^2 = 48$$

$$c^2 = \frac{48}{16} = 3$$

$c = \pm\sqrt{3}$ ← but $+\sqrt{3}$ is NOT IN $[-3, -1]$

ANS: $\boxed{-\sqrt{3}}$ (A)

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(11) $f(x) = x^3 - 4x^2 + 5$

$f'(x) = 3x^2 - 8x$

CRIT. PTS

$0 = 3x^2 - 8x$

$0 = x(3x-8)$

$x = 0 \quad x = 8/3$

$f'(x)$ DNE never

NO endpts

	$-\infty$		0		$8/3$		∞
x		$-$		$+$		$+$	
$3x-8$		$-$		$-$		$+$	
F'		$+$		$-$		$+$	
F		INC		Dec		INC	

$\boxed{\begin{array}{l} INC: (-\infty, 0) \cup (8/3, \infty) \\ Dec: (0, 8/3) \end{array}}$

(12) $y = -2\cot x \quad [-\pi, \pi]$

$y' = 2\csc^2 x = 2(\csc x)^2$

$y'' = 4(\csc x)'(-\csc x \cot x)$

$y'' = -4\csc^2 x \cot x$

$y'' = -4\left(\frac{1}{\sin^2 x}\right)\left(\frac{\cos x}{\sin x}\right) = \frac{-4\cos x}{\sin^3 x}$

$y'' = 0$

$-4\cos x = 0$

$\boxed{x = -\pi/2, \pi/2}$

y'' DNE

$\sin^3 x = 0$

$\sin x = 0$

$\boxed{x = 0, \pi, -\pi}$

end pts

$\boxed{x = \pi, -\pi}$

	$-\pi$		$-\pi/2$		0		$\pi/2$		π
$-4\cos x$		$+$		$-$		$-$		$+$	
$(\sin x)^3$		$-$		$-$		$+$		$+$	
y''		$-$		$+$		$-$		$+$	
$y(x)$		$\cap$		$\cup$		$\cap$		$\cup$	

$\boxed{\text{CON. UP } \left(-\frac{\pi}{2}, 0\right) \cup \left(\frac{\pi}{2}, \pi\right)}$

$\boxed{\text{CON Down } \left(-\pi, -\frac{\pi}{2}\right) \cup \left(0, \pi/2\right)}$

(A)

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(13) $y = -x^2 - 8x - 13$

$y' = -2x - 8$

$y'' = -2 \leftarrow$ so y'' is always $\ominus$
y is always concave down

$\boxed{\text{(D) No inflection point; Concave up: Never; Concave down: } (-\infty, \infty)}$

(14) $y = (-2x+8)^{1/2}$ $[-2, 2]$

$y' = \frac{1}{2}(-2x+8)^{-1/2}(-2) = -1(-2x+8)^{-1/2}$

$y' = \dfrac{-1}{\sqrt{-2x+8}}$

CRIT. PTS

$y' = 0$
$-1 = 0$
never

y' DNE
$\sqrt{-2x+8} = 0$
$-2x+8 = 0$
$-2x = -8$
$\boxed{x=4} \leftarrow$ $x=4$ is not in interval

end pts
$x = -2, 2$

	-2		2
-1		$-$	
$\sqrt{-2x+8}$		$+$	
y'		$\ominus$	
y	max	$\rightarrow$	min

$\boxed{\max\ (-2, 2\sqrt{3})}$

$y = \sqrt{-2(-2)+8} = \sqrt{12}$
$= 2\sqrt{3}$

$\boxed{\min\ (2, 2)}$

$y = \sqrt{-2(2)+8}$
$= \sqrt{4} = 2$

(C)

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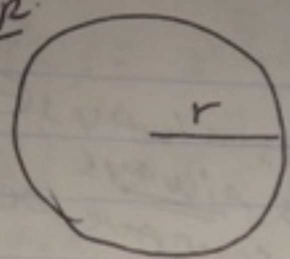
(15) DR:



$\frac{dA}{dT} = -81\pi \frac{\text{IN}^2}{\text{hr}}$

WANT $\frac{dr}{dT}$ when $r = 13$ in

E: $A = \pi r^2$

D: $\frac{dA}{dT} = 2\pi r \frac{dr}{dT}$

S: $-81\pi = 2\pi(13)\frac{dr}{dT}$

$$\frac{-81\pi}{2\pi(13)} = \frac{dr}{dT} = \frac{-81\pi}{26\pi} = \boxed{\frac{-81}{26}\ \text{IN/hr}}$$

(A)