CHAb - Ch. 5B review ANS (P.1)

① $\int -20x^3 \sec^2(x^4-5)\,dx = \int -20x^3 \sec^2 u \cdot \frac{du}{4x^3}$

$u = x^4 - 5$

$du = 4x^3\,dx$

$\frac{du}{4x^3} = dx$

$= \int -5\sec^2 u\,du$

$= -5\int \sec^2 u\,du$

$= -5[\tan u] + C$

C $= \boxed{-5\tan(x^4-5) + C}$

② $\int 10e^{2x}\csc(e^{2x}-5)\cot(e^{2x}-5)\,dx$

$u = e^{2x} - 5$

$du = 2e^{2x}\,dx$

$\frac{du}{2e^{2x}} = dx$

$= \int 10e^{2x}\csc u \cot u \cdot \frac{du}{2e^{2x}} = \int 5\csc u \cot u\,du$

$= 5\int \csc u \cot u\,du = 5[-\csc u] + C$

$= \boxed{-5\csc(e^{2x}-5) + C}$ A

③ $\int \frac{-4x}{x^2-3}\,dx = \int \frac{-4x}{u}\cdot\frac{du}{2x} = -\frac{4}{2}\int \frac{du}{u} = -2\int \frac{du}{u}$

$u = x^2 - 3$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$= -2\ln|u| + C$

$= \boxed{-2\ln|x^2-3| + C}$ C

CHAb - ch.5B review ANS (p.2)

(4) $\int 15x^4 e^{x^5 - 2}\,dx = \int 15x^4 e^u \cdot \frac{du}{5x^4} = \frac{15}{5}\int e^u\,du$

$u = x^5 - 2$
$du = 5x^4\,dx$
$\frac{du}{5x^4} = dx$

$= 3\int e^u\,du = 3e^u + c$

$= \boxed{3e^{x^5-2} + c}$

(A)

(5) $\int \frac{-2 \cdot 4^{-1+\ln(-4x)}}{x}\,dx = \int \frac{-2 \cdot 4^u}{x}\,x\,du$

3 possible "u's":

(1) Denom $u = x$ ← not helpful, just changes x's to u's

(2) $-4x = u$ (inside of ln)... not helpful enough b/c you still have a super messy integral

(3) whole exponent

$u = -1 + \ln(-4x)$

$du = \frac{1}{-4x} \cdot \frac{-4}{dx} = \frac{1}{x}\,dx$

$x\,du = dx$

this is the winner

$= \int -2 \cdot 4^u\,du$

$= -2\int 4^u \cdot du$

$= -2\left(\frac{1}{\ln 4}\right) \cdot 4^u + c$

$= \boxed{-2\,\frac{1}{\ln 4} \cdot 4^{-1+\ln(-4x)} + c}$

$= \boxed{\frac{-2 \cdot 4^{-1+\ln(-4x)}}{\ln 4} + c}$

RECALL: $\int a^u\,du = \frac{1}{\ln a} \cdot a^u + c$

(A)

(6) $\int \frac{1}{x(2+\ln(-4x))}\,dx = \int \frac{1}{x\,u} \cdot x\,du = \int \frac{1}{u}\,du$

the $(2+\ln(-4x))$ is in parentheses for a reason... it's likely your "u"
Because if you make the entire Denom "u" then you have a product rule which makes it messier

$u = 2 + \ln(-4x)$

$du = \frac{1}{-4x} \cdot -4\,dx \rightarrow du = \frac{1}{x}\,dx$

$x\,du = dx$

$= \ln|u| + c$

$= \boxed{\ln|2 + \ln(-4x)| + c}$

(C)

CHAB ch.5B review solns (p.3)

(7) $\int 10x^4(2x^5+5)^5 dx = \int 10x^4 u^5 \cdot \frac{du}{10x^4} = \int u^5 du$

$u = 2x^5+5$
$du = 10x^5 dx$
$\frac{du}{10x^5} = dx$

$= \frac{u^6}{6} + C = \boxed{\frac{1}{6}(2x^5+5)^6 + C}$

(C)

(8) $\int 20x^3(x^4+3)^{-5} dx = \int 20x^3 u^{-5} \cdot \frac{du}{4x^3} = \frac{20}{4}\int u^{-5} du$

$u = x^4+3$
$du = 4x^3 dx$
$\frac{du}{4x^3} = dx$

$= 5\int u^{-5} du = 5\frac{u^{-4}}{-4} + C$

$= \frac{-5}{4(u^4)} + C = \boxed{\frac{-5}{4(x^4+3)^4} + C}$

(D)

(9) $\int 10e^{5x}\sqrt{e^{5x}+1}\, dx = \int 10e^{5x}\sqrt{u} \cdot \frac{du}{5e^{5x}} = \frac{10}{5}\int \sqrt{u}\, du$

Possible "u's"

(1) $e^{5x}+1 = u$ ← inside the root and it's derivative appears elsewhere in integrand

yes!!

$u = e^{5x}+1$
$du = e^{5x} \cdot 5dx$
$\frac{du}{5e^{5x}} = dx$

$= 2\int u^{1/2} du$

$= 2\frac{u^{3/2}}{3/2} + C$

$= 2 \cdot \frac{2}{3} u^{3/2} + C$

(2) $u = 5x$ ← not enough to make integrand look like simple antiderivative

NO!

$\boxed{= \frac{4}{3}(e^{5x}+1)^{3/2} + C}$ (C)

(10) $\int \frac{2(-2+\ln(-2x))^{-5}}{x} dx = \int \frac{2u^{-5}}{x} \cdot x\, du = 2\int u^{-5} du$

much like #5 and #6, it makes sense to pick the most complicated "inside fxn" as our "u"

$u = -2 + \ln(-2x)$
$du = \frac{1}{-2x} \cdot -2\, dx = \frac{1}{x} dx$

$x\, du = dx$

$= 2\frac{u^{-4}}{-4} + C = -\frac{1}{2}u^{-4} + C$

$= \frac{-1}{2u^4} + C$

$= \boxed{-\frac{1}{2(-2+\ln(-2x))^4} + C}$

(D)

CHAb-Ch.5B review solns (p.4)

(11) $\int_0^1 \frac{-12x}{(2x^2+2)^2}dx = \int_2^4 \frac{-12x}{u^2}\cdot\frac{du}{4x} = \boxed{\int_2^4 \frac{-3du}{u^2}}$ (D)

$u=2x^2+2$

$du=4x\,dx$

$\frac{du}{4x}=dx$

TOP BOUND: $x=1$

$u=2(1)^2+2=4$

BOTTOM BOUND: $x=0$

$u=2(0)^2+2=2$

(12) $\int_0^1 \frac{12x}{(2x^2+3)^2}dx = \int_3^5 \frac{12x}{u^2}\cdot\frac{du}{4x} = \boxed{\int_3^5 \frac{3du}{u^2}}$

$u=2x^2+3$

$du=4x\,dx$

$\frac{du}{4x}=dx$

TOP: $x=1$

$u=2(1)^2+3=5$

BOTTOM: $x=0$

$u=2(0)^2+3=3$

(A)

(13) $\int_{-3}^0 \frac{-2x}{(x^2+1)^2}dx = \left[\frac{1}{x^2+1}\right]_{-3}^0 = \frac{1}{1}-\frac{1}{9+1} = \frac{1}{1}-\frac{1}{10} = \frac{10}{10}-\frac{1}{10}$

$=\boxed{\frac{9}{10}}$ (A)

WORK: (w/o bounds)

$\int \frac{-2x}{(x^2+1)^2}dx = \int \frac{-2x}{u^2}\frac{du}{2x}$

$u=x^2+1$

$du=2x\,dx$

$\frac{du}{2x}=dx$

$=-1\int u^{-2}du$

$=\left[\frac{-1u^{-1}}{-1}\right]$

$=\left[\frac{1}{u}\right]=\left[\frac{1}{x^2+1}\right]$

(14) $\int_0^2 \frac{12x}{(3x^2+4)^2}dx$

$=\left[\frac{-2}{(3x^2+4)}\right]_0^2$

$=\frac{-2}{3(4)+4}-\frac{-2}{3(0)+4}$

$=\frac{-2}{16}-\frac{-2}{4}$

$=-\frac{1}{8}+\frac{1}{2}=\frac{-1}{8}+\frac{4}{8}$

$=\boxed{3/8}$ (B)

WORK (w/o BOUNDS)

$u=3x^2+4$

$du=6x\,dx$

$\frac{du}{6x}=dx$

$\int \frac{12x}{(3x^2+4)^2}dx=$

$=\int \frac{12x}{u^2}\cdot\frac{du}{6x}$

$=2\int u^{-2}du$

$=\left[2\frac{u^{-1}}{-1}\right]$

$=\left[\frac{-2}{u}\right]=\left[\frac{-2}{3x^2+4}\right]$

(15) Avg value of $f(x)$ on $[a,b]$

$=\frac{1}{b-a}\int_a^b f(x)dx$

so

$\frac{1}{4-0}\int_0^4 -2x+2\,dx$

$\frac{1}{4}\int_0^4 -2x+2\,dx$

$=\frac{1}{4}\left[-x^2+2x\right]_0^4$

$=\frac{1}{4}\left[(-16+8)-(0+0)\right]=\frac{1}{4}(-8)=\frac{-2}{1}=\boxed{-2}$ (C)

CHAB-ch.5B review SO/N p.5

(16) Avg value of $f(x)$ on $[a,b]$ is

$$\frac{1}{b-a}\int_a^b f(x)\,dx \quad \text{so...} \quad \frac{1}{3-0}\int_0^3 3x^{1/2}\,dx$$

$$=\left[\frac{1}{3}\left(3\frac{x^{3/2}}{3/2}\right)\right]_0^3 = \left[\frac{1}{3}\cdot 3\cdot\frac{2}{3}x^{3/2}\right]_0^3$$

$$=\frac{2}{3}\left[x^{3/2}\right]_0^3 = \frac{2}{3}3^{3/2} - \frac{2}{3}0^{3/2} = \frac{2}{3}3^{3/2}$$

$$=\frac{2}{3}\sqrt{3^3} = \frac{2}{3}\sqrt{27} = \frac{2}{3}\sqrt{9\cdot 3} = \frac{2}{\cancel{3}}\cancel{3}\sqrt{3}$$

$$=\boxed{2\sqrt{3}} \quad \text{D}$$

(17) $$\frac{1}{1-0}\int_0^1 -x^{1/2}\,dx = 1\left[\frac{-x^{3/2}}{3/2}\right]_0^1 = \left[\frac{-2}{3}x^{3/2}\right]_0^1$$

$$= -\frac{2}{3}(1)^{3/2} - \frac{2}{3}(0) = \boxed{\frac{-2}{3}} \quad \text{B}$$

(18) $$F(x) = \int_{-1}^{x} T^2 - 4T - 1\,dT$$

$$F'(x) = \frac{d}{dx}\int_{-1}^{x} T^2 - 4T - 1\,dT$$

TOP BOUND $= T$ $\quad dT$ $\quad$ BOTTOM $= T$ $\quad dT$

$$= (x^2 - 4x - 1)(1) - (1+4-1)(0)$$

$$= \boxed{x^2 - 4x - 1} \quad \text{D}$$

(19) $$F(x) = \int_{-4}^{x^3} (-T^2 - 2T + 5)\,dT$$

$$F'(x) = \frac{d}{dx}\int_{-4}^{x^3} (-T^2 - 2T + 5)\,dT$$

$$= \left[-(x^3)^2 - 2(x^3) + 5\right](3x^2)$$

TOP $= T$ $\quad dT$

$$- \left(-(-4)^2 - 2(-4) + 5\right)(0)$$

BOT $= T$ $\quad dT$

$$\boxed{\begin{aligned} F'(x) &= 3x^2(-x^6 - 2x^3 + 5) \\ &= -3x^8 - 6x^5 + 15x^2 \end{aligned}} \quad \text{D}$$

~~(20) $F(x) = \int^{x^3} (-2T-1)\,dT$~~

CHAD - Ch.5B SOLNS (review) p.6

(20) $F(x) = \int_{x}^{x^2} (-2T-1)\,dT$ ← IT'S SQUARED (TOP OF PAG got cut off on mvc)

$$F'(x) = \frac{d}{dx}\int_{x}^{x^2} (-2T-1)\,dT$$

$$= \underbrace{(-2x^2-1)}_{TOP=T}\underbrace{(2x)}_{dT} - \underbrace{(-2x-1)}_{BOT=T}\underbrace{(1)}_{dT}$$

$$= -4x^3 - 2x + 2x + 1 = \boxed{-4x^3+1}$$

A

18) D ✓ 19) D ✓ 16) D ✓ 20) A ✓

CHAb - Ch.5B review SolNS - (p.8) FR

(D) "speeding up" means $A(T)$ and $V(T)$ have the same sign (either both ⊖ or both ⊕)

We already have a $V(T)$ sign chart (thanks part C!!)
So we just need an $A(T)$ sign chart.

① $A(T)=0$

$6T-16=0$

$T=\frac{16}{6}=\frac{8}{3}$

②

	0		$8/3$		∞
$6T-16$		−	0	+	
$A(T)$		⊖		⊕	

Now let's put the two sign charts together... the critical values (to go across the top) are 0 (endpt), $1/3$ (for $V(T)$), $8/3$ (for $A(T)$), 5 (for $V(T)$) and ∞ (endpt)

	0		$1/3$		$8/3$		5		∞
From part C → $V(T)$		+	0	−		−	0	+	
From Above → $A(T)$		−		−	0	+		+	
		opp		same		opp		same	

NOTE that "slowing down" means $A(T)$ and $V(T)$ have opposite signs

$$\boxed{\text{So "speeding up" at } \left(\tfrac{1}{3}, \tfrac{8}{3}\right) \cup (5, \infty)}$$

CHAb - ch.5B review solns - FR (p.7)

$v(T) = 3T^2 - 16T + 5$ FOR $T \geq 0$. POSITION at $T=1$ is 7.
(so $x(1) = 7$)

Ⓐ $v(T) = 3T^2 - 16T + 5$

$v'(T) = \boxed{A(T) = 6T - 16}$

Ⓑ $x(T) = \int v(T)\,dT = \int 3T^2 - 16T + 5\,dT = T^3 - 8T^2 + 5T + C$

$x(T) = T^3 - 8T^2 + 5T + C$

$x(1) = 7 = 1 - 8 + 5 + C$

$7 = -2 + C$

$9 = C$

$\boxed{x(T) = T^3 - 8T^2 + 5T + 9}$

Ⓒ moving right means $v(T) > 0$

① FIND when it's (the particle) AT REST (meaning $v(T) = 0$)

$v(T) = 0 = 3T^2 - 16T + 5$

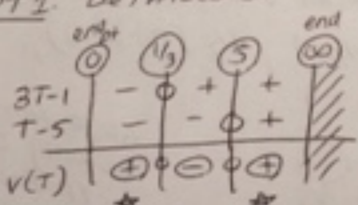
$0 = (3T - 1)(T - 5)$

$3T - 1 = 0 \quad T - 5 = 0$

$T = 1/3 \quad T = 5$

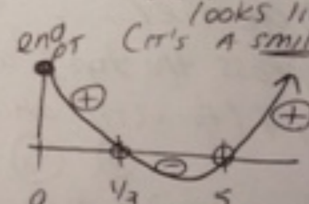
② NOW MAKE A SIGN CHART (NOTE $T \geq 0$ WAS GIVEN IN INTRO)

OPT 1: DETAILED SIGN chart



$\boxed{\text{ANS: } [0, 1/3) \cup (5, \infty)}$

OPT 2: knowledge of what $v(T) = 3T^2 - 16T + 5$ looks like (it's a smile)



$\boxed{\text{ANS: } [0, 1/3) \cup (5, \infty)}$

NOTE: $T=0$ is included b/c the particle is moving at $T=0$. $T = 1/3$ and $T = 5$ are NOT because the particle is AT REST at those times.