CHAb - Ch. 6 review SOLNS

① $\int_6^7 3\sqrt{x}\,dx = \left[3\left(\tfrac{2}{3}\right)x^{3/2}\right]_6^7$

$= \left[2\left(x^{3/2}\right)\right]_6^7 = \left[2x \cdot x^{1/2}\right]_6^7 = \left[2x\sqrt{x}\right]_6^7$

$= 2(7)\sqrt{7} - 2(6)\sqrt{6} = \boxed{14\sqrt{7} - 12\sqrt{6}}$ Ⓒ

② even though bounds are $x = -1, 3$ we should make sure the curves don't switch in the middle... so find the points of intersection

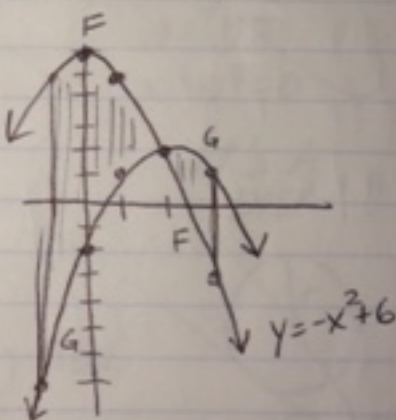
$-x^2 + 6 = -x^2 + 4x - 2$

$6 = 4x - 2$

$8 = 4x$

$x = 2$ ← so the curves will switch in the middle

x	F(x) $y = -x^2+6$	g(x) $y = -x^2+4x-2$
-1	$-1+6=5$	$-1-4-2=-7$
0	6	-2
1	$-1+6=5$	$-1+4-2=1$
2	$-4+6=2$	$-4+8-2=2$
3	$-9+6=-3$	$-9+12-2=1$



$= \int_{-1}^{2} (-x^2+6) - (-x^2+4x-2)\,dx + \int_{2}^{3} (-x^2+4x-2) - (-x^2+6)\,dx$

$= \int_{-1}^{2} -x^2+6+x^2-4x+2\,dx + \int_{2}^{3} -x^2+4x-2+x^2-6\,dx$

$= \int_{-1}^{2} -4x+8\,dx + \int_{2}^{3} 4x-8\,dx$ (continued on next page)

CHAb. – Ch. 6 review solns

(2) (cont.)

$$\int_{-1}^{2} -4x+8\,dx + \int_{2}^{3} 4x-8\,dx$$

$$= \left[-2x^2+8x\right]_{-1}^{2} + \left[2x^2-8x\right]_{2}^{3}$$

$$= \left[(-8+16)-(-2+8)\right] + \left[(18-24)-(8-16)\right]$$

$$= \left[8+10\right] + \left[-6+8\right] = 18+2 = \boxed{20}$$

(B)

---

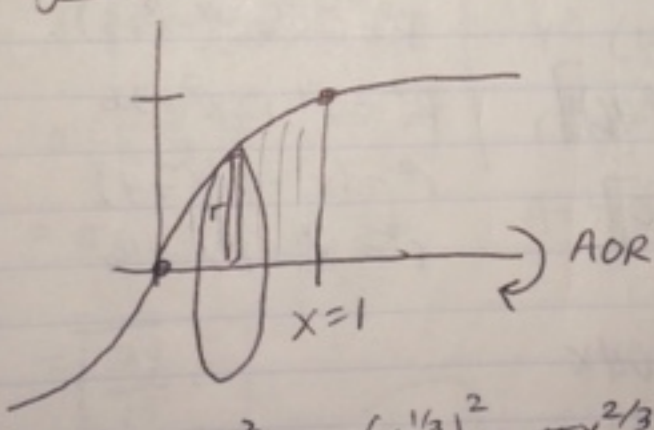
(3) $y=\sqrt[3]{x}$, $y=0$, $x=1$

<u>FIND other bound</u>: $y=0$ $y=\sqrt[3]{x}$

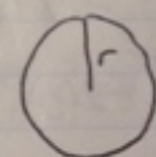
$0=\sqrt[3]{x}$

$x=0^3=0$

<u>graph</u>



IT'S A DISK!!



$r=\sqrt[3]{x}-0$

$r=x^{1/3}$

$$A=\pi r^2 = \pi\left(x^{1/3}\right)^2 = \pi x^{2/3}$$

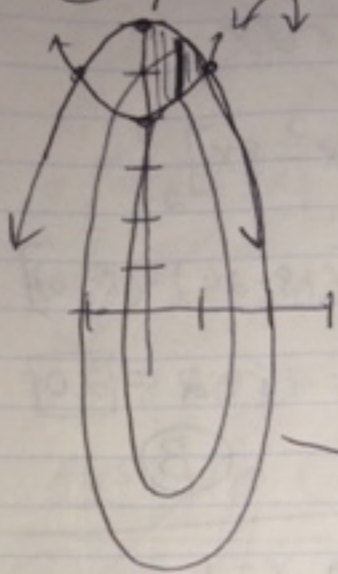
$$V=\pi\int_0^1 x^{2/3}\,dx = \pi\left[\tfrac{3}{5}x^{5/3}\right]_0^1$$

$$= \pi\left[\tfrac{3}{5}(1)^{5/3} - \tfrac{3}{5}(0)^{5/3}\right] = \boxed{\tfrac{3}{5}\pi}$$

(B)

CHAb - ch.6 review solns

④ $y=-x^2+6$, $y=x^2+4$, $\underbrace{x=0, x=1}_{\text{Bounds}}$



To grab, finding intersection helps (even though bounds are given)

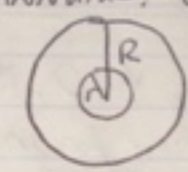
$-x^2+6=x^2+4$

$2=2x^2$

$1=x^2$

$x=\pm 1$

IT'S AN ANNULUS! (washer)



$R=-x^2+6-\cancel{0}$

$R=-x^2+6$

$r=x^2+4-\cancel{0}$

$r=x^2+4$

$A=\pi R^2-\pi r^2$

$A=\pi(R^2-r^2)$

$A=\pi\left[x^4-12x^2+36-(x^4+8x^2+16)\right]$

$A=\pi\left[-20x^2+20\right]$

$R^2=(-x^2+6)(-x^2+6)$

$R^2=x^4-12x^2+36$

$r^2=(x^2+4)(x^2+4)$

$r^2=x^4+8x^2+16$

$$V=\pi\int_0^1 -20x^2+20\,dx$$

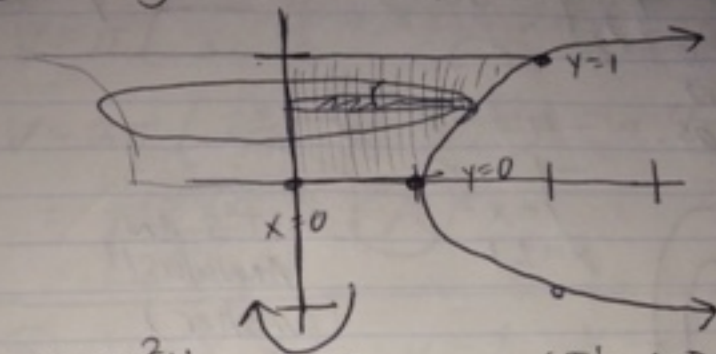
$$=\pi\left[\frac{-20}{3}x^3+20x\right]_0^1=\pi\left[\left(\frac{-20}{3}+20\right)-(0+0)\right]$$

$$=\pi\left(\frac{-20}{3}+\frac{60}{3}\right)=\boxed{\frac{40}{3}\pi}$$

D

CHAB – ch. 6 review solns

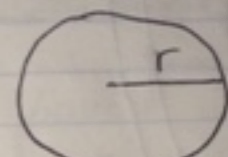
(5) $x=y^2+1,\ x=0,\ y=0,\ y=1$



y	$x=y^2+1$
-1	$1+1=2$
0	$0+1=1$
1	$1+1=2$

AOR

IT'S A DISK!!



$$r = \underbrace{y^2+1 - \emptyset}^{\text{right} - \text{left}}$$

$$r = y^2+1$$

$$r^2 = (y^2+1)(y^2+1)$$

$$r^2 = y^4+2y^2+1$$

$$A = \pi r^2 = \pi(y^4+2y^2+1)$$

$$V = \pi\int_0^1 y^4+2y^2+1\,dy$$

$$= \pi\left[\frac{y^5}{5} + \frac{2}{3}y^3 + y\right]_0^1$$

$$= \pi\left(\frac{1}{5} + \frac{2}{3} + 1 - (0+0+0)\right]$$

$$= \pi\left(\frac{3}{15} + \frac{10}{15} + \frac{15}{15}\right)$$

$$= \boxed{\frac{28}{15}\pi}$$

(C)

CHAD - ch.6 review sol/ns

⑥ $y=-4,\ y=-x^2-3$ AXIS: $y=1$

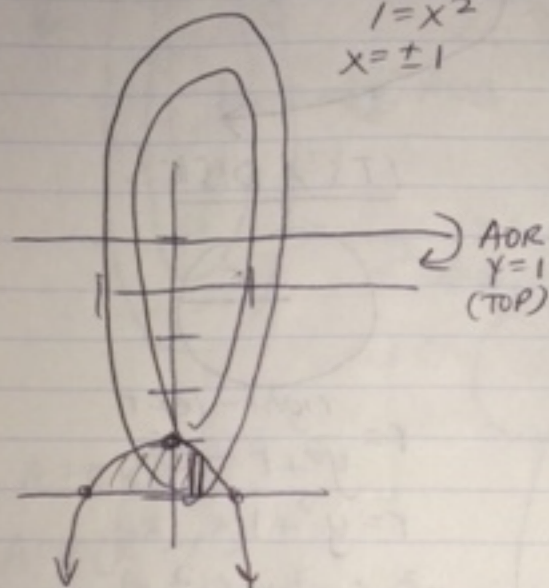
FIND BOUNDS: $-4=-x^2-3$

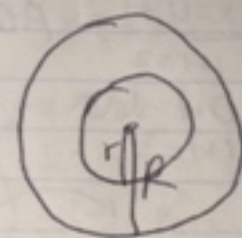
$-1=-x^2$

$1=x^2$

$x=\pm 1$

IT'S AN ANNULUS!! (WAsher)





TOP − BOT

$R = 1-(-4) = 5$

$R=5$

$r = 1-(-x^2-3)$

$r = 1+x^2+3$

$r=x^2+4$

$R^2=5^2=25$

$r^2=(x^2+4)(x^2+4)$

$r^2=x^4+8x^2+16$

$A=\pi R^2-\pi r^2$

$=\pi(R^2-r^2)$

$A=\pi\left(25-(x^4+8x^2+16)\right)$

$A=\pi\left(-x^4-8x^2+9\right)$

$V=\pi\int_{-1}^{1}-x^4-8x^2+9\,dx$ OR $2\pi\int_0^1 -x^4-8x^2+9\,dx$

(since BOUND AreA IS symmetric about $x=0$)

$=\pi\left[\frac{-x^5}{5}-\frac{8}{3}x^3+9x\right]_{-1}^{1}$

$=\pi\left[\left(-\frac{1}{5}-\frac{8}{3}+9\right)-\left(\frac{1}{5}+\frac{8}{3}-9\right)\right]=\pi\left(\frac{-2}{5}-\frac{16}{3}+18\right)$

CHAb - ch. 6 - review solns

(6) (cont.)

$V = \pi\left(\frac{-2}{5} - \frac{16}{3} + 18\right) = 2\pi\left(-\frac{1}{5} - \frac{8}{3} + 9\right)$

$V = 2\pi\left(-\frac{3}{15} - \frac{40}{15} + \frac{135}{15}\right) = 2\pi\left(\frac{92}{15}\right)$

$= \boxed{\frac{184\pi}{15}}$ (A)

---

(7) $\underbrace{y = x^2+1,\ y = 1}$, $x = 2$

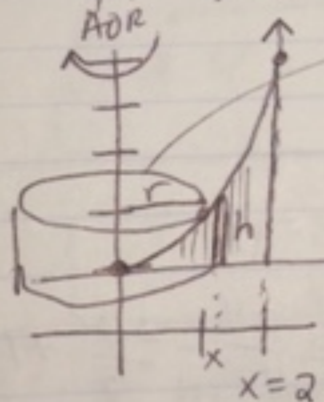
$\uparrow$ one bound

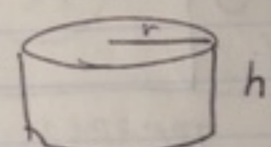
$1 = x^2+1$

$x^2 = 0$

$x = 0 \leftarrow$ other bound

IT'S A shell...





Right - left

$r = x - 0 = x$

Top - bot

$h = x^2+1-1 = x^2$

$A = 2\pi r h = 2\pi x(x^2) = 2\pi x^3$

$V = 2\pi\int_0^2 x^3\,dx = 2\pi\left[\frac{x^4}{4}\right]_0^2 = 2\pi\left[\frac{16}{4} - \frac{0}{4}\right]$

$\boxed{= 8\pi}$ (C)

CHAD - Ch. 6 review sol'ns

(8) $y = -\frac{x^2}{9} + 4, \; y = 0$

FIND bounds:

$0 = -\frac{x^2}{9} + 4$

$-4 = -x^2/9$

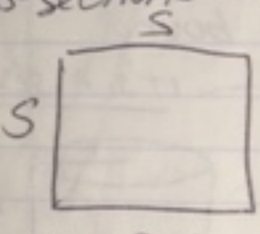
$-36 = -x^2$

$36 = x^2$

$x = \pm 6$

symmetric about $x = 0$!!

Cross-sections are SQUARES ($\perp$ to x-axis)



TOP - BOT

$S = -\frac{x^2}{9} + 4 - 0 = -\frac{x^2}{9} + 4$

$A = S^2 = \left(-\frac{x^2}{9} + 4\right)\left(-\frac{x^2}{9} + 4\right) = \frac{x^4}{81} - \frac{2x^2}{9} + 16$

$V = \int_{-6}^{6} \frac{x^4}{81} - \frac{2x^2}{9} + 16 \, dx$ OR $\underbrace{2\int_0^6 \frac{x^4}{81} - \frac{8x^2}{9} + 16 \, dx}_{\text{way easier!!}}$

$= 2\left[\frac{x^5}{\underset{(81\cdot5)}{405}} - \frac{8x^3}{(9\cdot3)} + 16x\right]_0^6$

$= 2\left[\frac{x^5}{405} - \frac{8x^3}{27} + 16x\right]_0^6$

continued on next page...

CHAD - ch. 6 review soln's

(8) (cont.)

$$V = 2\left[\frac{x^5}{405} - \frac{\cancel{8}x^3}{27} + 16x\right]_0^6$$

$$= 2\left[\underbrace{\frac{6^5}{405} - \frac{\cancel{8}(6^3)}{27} + 16(6)}_{\text{great time to use your calculator for arithmetic!!}} - (0+0+0)\right]$$

$$= 2\left[\frac{7776}{405} - \frac{1728}{27} + 96\right]$$

$$= \boxed{102.4 = \frac{512}{5}}$$

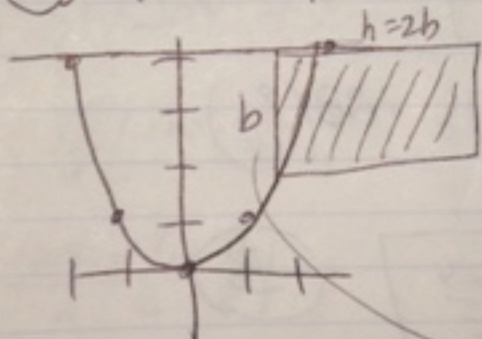
A

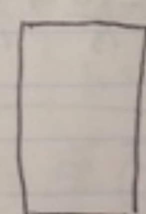
(9) $y = 4$ $\quad y = x^2$

BOUNDS

$x^2 = 4$

$x = \pm 2$





$h = 2b$
$= 2(4-x^2)$

$b = 4 - x^2$
TOP - bott

$A = b \cdot h = b \cdot 2b$

$A = 2b^2$

$A = 2(4-x^2)^2$

$= 2(4-x^2)(4-x^2)$

$= 2(16 - 8x^2 + x^4) = 2x^4 - 16x^2 + 32$

CHAD - Ch. 6 review SOLNS

(9) (cont.)

$A = 2x^4 - 16x^2 + 32$

b/c BAse Area is symmetric about $x=0$

$V = \int_{-2}^{2} 2x^4 - 16x^2 + 32\,dx \overset{OR}{=} \underbrace{2\int_{0}^{2} 2x^4 - 16x^2 + 32\,dx}_{\text{easier to plug-in}}$

$V = 2\left[\frac{2}{5}x^5 - \frac{16}{3}x^3 + 32x\right]_0^2$

$= 2\left[\frac{2}{5}(32) - \frac{16}{3}(8) + 32(2) - (0+0+0)\right]$

$= 2(16)\left[\frac{2}{5}(2) - \frac{8}{3} + 2(2)\right]$

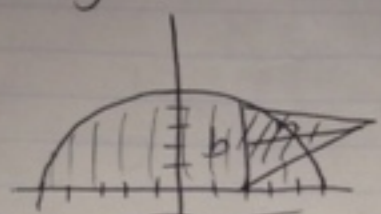
$= 32\left[\frac{4}{5} - \frac{8}{3} + 4\right] = 32(4)\left[\frac{1}{5} - \frac{2}{3} + 1\right]$

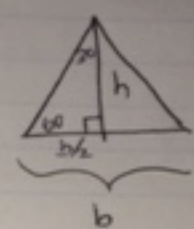
$= 128\left[\frac{3}{15} - \frac{10}{15} + \frac{15}{15}\right] = 128\left(\frac{8}{15}\right) =$

$= \boxed{68.2\overline{66} = \frac{1024}{15}}$ B

CHAb – Ch. 6 review soln's

(10) $y=\sqrt{25-x^2}$ ← TOP hALF of a circle w/ $r=5$ and center at $(0,0)$





base Area is symmetric about $x=0$

$$V=\int_{-5}^{5}\frac{\sqrt{3}}{4}(25-x^2)\,dx$$

OR

$$V=2\int_{0}^{5}\frac{\sqrt{3}}{4}(25-x^2)\,dx$$

$$=\frac{2\sqrt{3}}{4}\left[25x-\frac{x^3}{3}\right]_0^5$$

$$=\frac{\sqrt{3}}{2}\left[25(5)-\frac{125}{3}-(0+0)\right]$$

$$=\frac{\sqrt{3}}{2}\left[125-\frac{125}{3}\right]$$

$$=\frac{125\sqrt{3}}{2}\left[1-\frac{1}{3}\right]$$

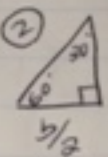
$$=\frac{125\sqrt{3}}{\cancel{2}}\left[\frac{\cancel{2}}{3}\right]$$

$$=\boxed{\frac{125\sqrt{3}}{3}}\approx 72.169$$

(D)

30°-60°-90° Δ ...

(2)



$h(\sqrt{3})$

$$\frac{1}{\sqrt{3}}=\frac{b/2}{h}$$

(1) $$\boxed{h=\frac{\sqrt{3}\,b}{2}}$$

$$A=\frac{1}{2}b\cdot h=\frac{1}{2}b\cdot\frac{\sqrt{3}}{2}b$$

$$\boxed{A=\frac{\sqrt{3}}{4}b^2}$$

$$b=\underset{\text{TOP}}{\sqrt{25-x^2}}-\underset{\text{BOT}}{\emptyset}$$

$$b=\sqrt{25-x^2}$$

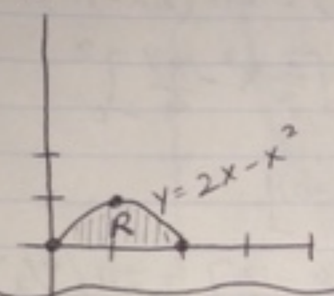
$$A=\frac{\sqrt{3}}{4}\left(\sqrt{25-x^2}\right)^2$$

$$\boxed{A=\frac{\sqrt{3}}{4}(25-x^2)}$$

CHAB - ch.6 review - SOLNS

(11) QI : $y = 2x - x^2$, x-AXIS $(y=0)$

graph:



bounds:

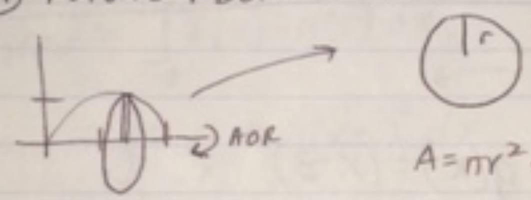
$0 = 2x - x^2$

$0 = x(2-x)$

$\boxed{x=0}$ $\quad 2 - x = 0$

$\boxed{x=2}$

(A) rotate About x-AXIS



IT'S A DISK!

$r = \underset{\text{TOP}}{2x - x^2} - \underset{\text{BOT}}{0}$

$r = 2x - x^2$

$A = \pi r^2 = \pi(2x - x^2)^2$

$A = \pi(2x - x^2)(2x - x^2)$ FOIL

$A = \pi(4x^2 - 2x^3 - 2x^3 + x^4) = \pi(x^4 - 4x^3 + 4x^2)$

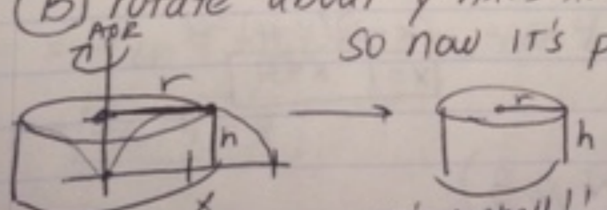
$V = \pi \int_0^2 x^4 - 4x^3 + 4x^2 \, dx = \pi \left[\frac{x^5}{5} - x^4 + \frac{4}{3}x^3 \right]_0^2$

$V = \pi \left(\frac{32}{5} - 16 + \frac{32}{3} - \emptyset \right) = 16\pi \left(\frac{2}{5} - 1 + \frac{2}{3} \right)$

$V = 16\pi \left(\frac{6 - 15 + 10}{15} \right) = 16\pi \left(\frac{1}{15} \right) = \boxed{\frac{16\pi}{15}}$

(B) rotate about y-AXIS ... slice is still "dx"

So now IT's parallel to the AoR



IT's A shell!!

$r = \underset{\text{RIGHT}}{x} - \underset{\text{LEFT}}{0} = \boxed{x = r}$

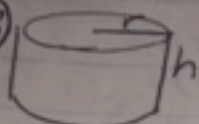
$h = \underset{\text{TOP}}{2x - x^2} - \underset{\text{BOT}}{0}$

$\boxed{h = 2x - x^2}$

continued ON page 2...

CHAb – ch. 6 review sol'ns

(11B)



$r = x$

$h = 2x - x^2$

$A = 2\pi r h = 2\pi(x)(2x - x^2) = 2\pi(2x^2 - x^3)$

$V = 2\pi \int_0^2 2x^2 - x^3 \, dx = 2\pi \left[\frac{2}{3}x^3 - \frac{x^4}{4} \right]_0^2$

$V = 2\pi \left[\left(\frac{2}{3}(8) - \frac{16}{4} \right) - (0 - 0) \right]$

$V = 2\pi \left[\frac{16}{3} - 4 \right] = 2\pi \left[\frac{16}{3} - \frac{12}{3} \right] = 2\pi(4/3)$

$\boxed{V = \frac{8\pi}{3}}$

(12) $f(x) = \frac{4}{x}$ $g(x) = (x-3)^2$

① FIND BOUNDS: $\frac{4}{x} = (x-3)^2 = (x-3)(x-3)$

$\frac{4}{x} = x^2 - 6x + 9$

$4 = x^3 - 6x^2 + 9x$

$0 = x^3 - 6x^2 + 9x - 4$

won't factor by: gcf, bi, Q.T, grouping... so use syn. division

$\boxed{x=1}$

$$\begin{array}{r|rrrr} 1 & 1 & -6 & 9 & -4 \\ & & 1 & -5 & 4 \\ \hline & 1 & -5 & 4 & \boxed{0} \end{array}$$

$x^2 - 5x + 4 = 0$

$(x-1)(x-4) = 0$

$\boxed{x = 1 \quad x = 4}$

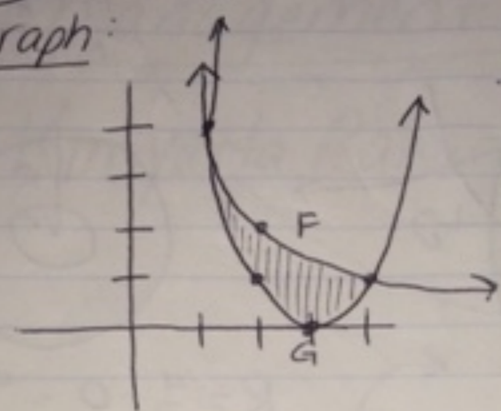
so bounds are $x = 1$ and $x = 4$.

(continued on p. 3)

CHAb - ch.6 review solns

(12A) BOUNDS: $x = 1, 4$

graph:



x	$f(x)=\frac{4}{x}$	$g(x)=(x-3)^2$
1	$4/1=4$	$(-2)^2=4$
2	$4/2=2$	$(-1)^2=1$
3	$4/3$	$(0)^2=0$
4	$4/4=1$	$(1)^2=1$

(A)

$$\text{Area} = \int_1^4 f(x) - g(x)\,dx$$

$$(x-3)(x-3)$$
$$x^2-6x+9$$

$$= \int_1^4 \frac{4}{x} - (x-3)^2\,dx$$

$$= \int_1^4 \frac{4}{x} - x^2 + 6x - 9\,dx$$

$$= \left[4\ln|x| - \frac{x^3}{3} + 3x^2 - 9x\right]_1^4$$

$$= \left(4\ln 4 - \frac{64}{3} + 48 - 36\right) - \left(\underbrace{4\ln 1}_{\emptyset} - \frac{1}{3} + \underbrace{3-9}_{(-6)}\right)$$

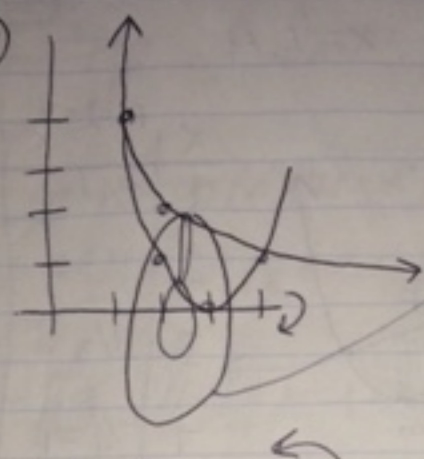
$$= 4\ln 4 - \frac{64}{3} + 12 + \frac{1}{3} + 6$$

$$= 4\ln 4 - \frac{63}{3} + 18 = 4\ln 4 - 21 + 18$$

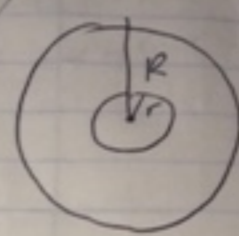
$$= \boxed{4\ln 4 - 3}$$

CHAD - ch. 6 review sol'ns

(12B)



IT'S A WASHER!!



$R = \frac{4}{x} - 0 = 4/x$

$r = (x-3)^2 - 0$

$r = (x-3)^2$

$r = x^2 - 6x + 9$

$A = \pi R^2 - \pi r^2$

$A = \pi (R^2 - r^2)$

$R^2 = \left(\frac{4}{x}\right)^2 = \frac{16}{x^2}$

$r^2 = (x^2 - 6x + 9)^2$

$= (x^2 - 6x + 9)(x^2 - 6x + 9)$

$r^2 = x^4 - 6x^3 + 9x^2$

$-6x^3 + 36x^2 - 54x$

$+9x^2 - 54x + 81$

$r^2 = x^4 - 12x^3 + 54x^2 - 108x + 81$

$$V = \pi \int_1^4 \frac{16}{x^2} - x^4 + 12x^3 - 54x^2 + 108x - 81 \, dx$$

$(16x^{-2})$

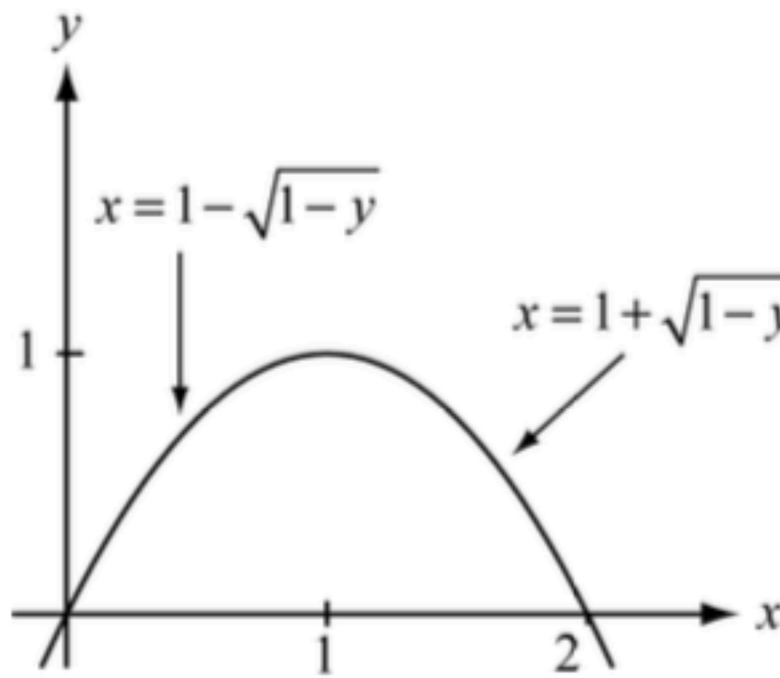
$$= \pi \left[-16/x - \frac{x^5}{5} + 3x^4 - 18x^3 + 54x^2 - 81x \right]_1^4$$

$$= \pi \left[\frac{-16}{4} - \frac{4^5}{5} + 3(4^4) - 18(4^3) + 54(16) - 81(4) + \frac{16}{1} + \frac{1}{5} - 3 + 18 - 54 + 81 \right]$$

$$= \pi \left(-4 - \frac{1024}{5} + 768 - 1152 + 864 - 324 + 16 + 1/5 - 3 + 18 - 54 + 81 \right) =$$

$$\pi \left(210 - \frac{1023}{5} \right) = \pi \left(\frac{1050 - 1023}{5} \right) = \boxed{\frac{27\pi}{5}}$$

*these numbers got super big [illegible]



(a) $\text{Volume} = \pi\int_0^2 (2x-x^2)^2\,dx = \pi\int_0^2 (4x^2-4x^3+x^4)\,dx$

$$= \pi\left(\frac{4}{3}x^3 - x^4 + \frac{1}{5}x^5\right)\Bigg|_0^2 = \frac{16}{15}\pi$$

(b) Shells:

$$\text{Volume} = 2\pi\int_0^2 xy\,dx = 2\pi\int_0^2 x(2x - x^2)\,dx = 2\pi\int_0^2 (2x^2 - x^3)\,dx$$

$$= 2\pi\left(\frac{2}{3}x^3 - \frac{1}{4}x^4\right)\Big|_0^2 = 2\pi\left(\left(\frac{16}{3} - \frac{16}{4}\right) - 0\right) = \frac{8\pi}{3}$$

Disks:

$$\text{Volume} = \pi\int_0^1 ((1 + \sqrt{1-y})^2 - (1 - \sqrt{1-y})^2)\,dy$$

$$= \pi\int_0^1 ((1 + 2\sqrt{1-y} + 1 - y) - (1 - 2\sqrt{1-y} + 1 - y))\,dy$$

$$= \pi\int_0^1 4\sqrt{1-y}\,dy$$

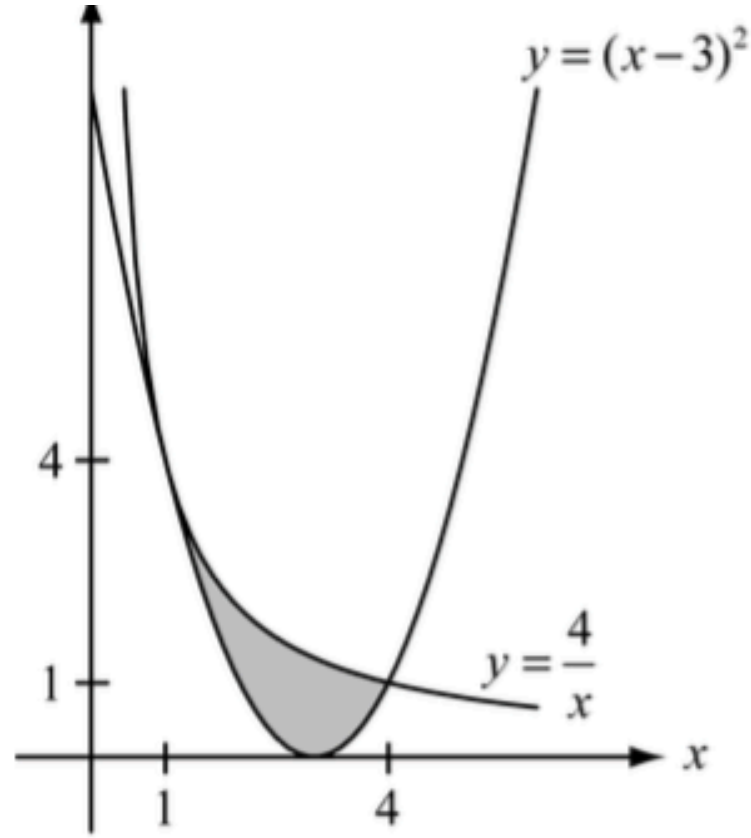
$$= -4\pi\left(\frac{2}{3}\right)(1-y)^{3/2}\Big|_0^1 = -\frac{8\pi}{3}(0-1) = \frac{8\pi}{3}$$

Intersection points occur when

$$\frac{4}{x}=(x-3)^2$$

$$0=x^3-6x^2+9x-4=(x-4)(x-1)^2$$

Thus the intersection points are at $(1,4)$ and $(4,1)$.



(a) Area $=\int_1^4\left(\frac{4}{x}-(x-3)^2\right)dx$

$$=\left(4\ln x-\frac{(x-3)^3}{3}\right)\Bigg|_1^4=4\ln 4-3$$

(b) Volume $=\int_1^4\pi\left(\left(\frac{4}{x}\right)^2-(x-3)^4\right)dx$

$$=\pi\left(-\frac{16}{x}-\frac{(x-3)^5}{5}\right)\Bigg|_1^4=\pi\left(\left(-4-\frac{1}{5}\right)-\left(-16-\frac{-32}{5}\right)\right)$$

$$=\frac{27\pi}{5}$$