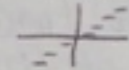
ChAb - ch. 7/8 review solns

① the slopes are always the same along a given vertical line, so they depend only on x, not y... thus IT MUST be Ⓓ $\frac{dy}{dx} = -x$

Further proof, when $x=0$ (on y-axis) $m = \frac{dy}{dx} = 0$ (horizontal lines)

② the slopes are level along the line $y=x$  ← this line... so

$\frac{dy}{dx} = 0$ when $y=x$, this means
$0 = y-x$ or $0 = x-y$... so IT'S either A or D.

PICK ANY EASY POINT, at $(1,0)$ the slope is ⊕. thus we can test both A and D to see which is correct.

A. $\frac{dy}{dx} = 1-0 = 1$ ← ⊕ :) D. $\frac{dy}{dx} = 0-1 = -1$ ← no! :(

SO IT'S Ⓐ

③ $\frac{dy}{dx} = 4x-3$

$\int dy = \int (4x-3)\,dx$

$\boxed{y = 2x^2 - 3x + c}$

Ⓐ

④ $\frac{dy}{dx} = 2\sin x$

$\int dy = \int 2\sin x\,dx$

$\boxed{y = -2\cos x + c}$

Ⓒ

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(5) $\frac{dy}{dx} = 2x - 1 \quad y(-1) = -1$

$\int dy = \int (2x-1)\,dx$

$y = x^2 - x + C$

$-1 = (-1)^2 - (-1) + C$

$-1 = 1 + 1 + C$

$-3 = C$

$\boxed{y = x^2 - x - 3}$

(B)

*Note: these are not clever answer choices... even w/o finding "c" the only Answer of the form

$y = x^2 - x + C$ was (B)

(6) $\frac{dy}{dx} = -2\sin x \quad y(0) = 3$

$\int dy = \int -2\sin x\,dx$

$y = 2\cos x + C$

$3 = 2\cos(0) + C$

$3 = 2 + C$

$1 = C$

$\boxed{y = 2\cos x + 1}$ (C)

but, again, even without finding "c," the only answer of the form

$y = 2\cos x + C$ is (C)

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⑦ $\frac{dy}{dx} = 3e^{x-y} = 3\frac{e^x}{e^y}$ ← by rules of exponents recall $\frac{b^x}{b^y} = b^{x-y}$

$\frac{dy}{dx} = \frac{3e^x}{e^y}$

$\int e^y dy = \int 3e^x dx$

$\boxed{e^y = 3e^x + c}$ ← ANS not solved for y

$\boxed{y = \ln(3e^x + c)}$ ← ANS solved for y (B)

---

⑧ $f(x) = \sin^{-1}(-4x^5)$ ← $u = -4x^5$, $du = -20x^4$

$f'(x) = \frac{-20x^4}{\sqrt{1-(-4x^5)^2}} = \boxed{\frac{-20x^4}{\sqrt{1-16x^{10}}}}$ (D)

but the only answer of the form $\frac{du}{\sqrt{1-u^2}}$ is (D)... so if you know the Derivative of arcsin u, it HAS to be D.

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⑨ $f(x) = \tan^{-1}(-4x^2)$ ← $u = -4x^2$ $du = -8x$

$f'(x) = \frac{-8x}{1+(-4x^2)^2} = \frac{-8x}{1+16x^4} = \boxed{\frac{-8x}{16x^4+1}}$ (C)

again, this is the only choice w/ $\frac{du}{1+u^2}$

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(10) $f(x) = \sec^{-1}(2x^3)$

$u = 2x^3$
$du = 6x^2$

$$f'(x) = \frac{6x^2}{|2x^3|\sqrt{(2x^3)^2-1}} = \boxed{\frac{6x^2}{|2x^3|\sqrt{4x^6-1}}}$$

(A)

---

(11) $$\int \frac{10x^4}{\sqrt{16-4x^{10}}}\,dx = \int \frac{\cancel{10x^4}}{\sqrt{a^2-u^2}} \cdot \frac{du}{\cancel{10x^4}}$$

$a^2 = 16$
$a = 4$

$u^2 = 4x^{10}$
$u = 2x^5$
$du = 10x^4\,dx$
$\frac{du}{10x^4} = dx$

$$= \int \frac{du}{\sqrt{a^2-u^2}}$$

$$= \arcsin\frac{u}{a} + c$$

(C)

* Also, this is the only $\sin^{-1}$ choice

$$= \boxed{\arcsin\frac{2x^5}{4} + c}$$

---

(12) $$\int \frac{10x^4}{1+4x^{10}}\,dx = \int \frac{\cancel{10x^4}}{a^2+u^2} \cdot \frac{du}{\cancel{10x^4}}$$

$a^2 = 1$
$a = 1$

$u^2 = 4x^{10}$
$u = 2x^5$
$du = 10x^4\,dx$
$\frac{du}{10x^4} = dx$

$$= \int \frac{du}{a^2+u^2}$$

$$= \frac{1}{a}\arctan\frac{u}{a} + c$$

$$= \frac{1}{1}\arctan\left(\frac{2x^5}{1}\right) + c$$

$$= \boxed{\arctan(2x^5) + c}$$

(D)

* also, it had to be C or D from the start b/c they are the only $\tan^{-1}$ choices

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(13) $\int \frac{16x^3}{4x^4\sqrt{16x^8-1}}\,dx$ ← IT has to be an arcsec... so IT's A or B...

$a^2=1$

$a=1$

$u^2=16x^8$

$u=4x^4$

$du=16x^3dx$

$\frac{du}{16x^3}=dx$

$$\int \frac{16x^3}{u\sqrt{u^2-a^2}}\cdot\frac{du}{16x^3} = \int \frac{du}{u\sqrt{u^2-a^2}}$$

$$= \frac{1}{a}\,\mathrm{arcsec}\frac{|u|}{a}+C = \frac{1}{1}\,\mathrm{arcsec}\frac{|4x^4|}{1}+C$$

$$= \boxed{\mathrm{arcsec}|4x^4|+C}$$

(B)

(14) $\int \frac{2e^{2x}}{\sqrt{4-e^{4x}}}\,dx = \int \frac{2e^{2x}}{\sqrt{a^2-u^2}}\cdot\frac{du}{2e^{2x}}$

$a^2=4$

$a=2$

$u^2=e^{4x}$

$u=e^{2x}$

$du=2e^{2x}dx$

$\frac{du}{2e^{2x}}=dx$

$$= \int\frac{du}{\sqrt{a^2-u^2}}$$

$$= \arcsin\left(\frac{u}{a}\right)+C$$

$$= \boxed{\arcsin\left(\frac{e^{2x}}{2}\right)+C}$$

(C)

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(15) $\displaystyle\int \frac{2e^{2x}}{4+e^{4x}}\,dx$ ← no root... so it's an arctan.. so it's (B)

BUT let's FINISH IT ANYWAY.

$4=a^2$ $\quad u^2=e^{4x}$

$a=2$ $\quad u=e^{2x}$

$du=2e^{2x}dx$

$\frac{du}{2e^{2x}}=dx$

$$\int \frac{2e^{2x}}{a^2+u^2}\cdot\frac{du}{2e^{2x}} = \int\frac{du}{a^2+u^2}$$

$$=\frac{1}{a}\arctan\frac{u}{a}+C = \boxed{\frac{1}{2}\arctan\frac{e^{2x}}{2}+C}$$

(B)

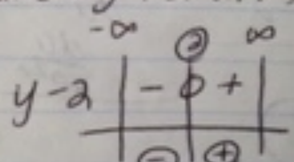
(16) (B) IF slopes are negative then $\frac{dy}{dx}<0$

so $x^4(y-2)<0$

so FIND zeros and make sign chart.

$x^4=0$ $\quad y-2=0$

$x=0$ $\quad y=2$



except at zero x^4 is always ⊕ b/c it's an even power

so $\frac{dy}{dx}<0$ when $y<2$

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16c $\frac{dy}{dx} = x^4(y-2)$

$$\frac{dy}{y-2} = x^4 dx$$ ← ① separate the variables

$$\int \frac{dy}{y-2} = \int x^4 dx$$ ← ② now integrate both sides and only put +C on the x-side

$u = y-2$
$du = dy$

$$\int \frac{dy}{u} = \frac{x^5}{5} + C$$

$$\ln|u| = \frac{x^5}{5} + C$$

$$\ln|y-2| = \frac{x^5}{5} + C$$ ← integrated

③ Now solve for y.

new C_1

$$\cancel{e}^{\cancel{\ln}|y-2|} = e^{\frac{x^5}{5} + C} = e^{x^5/5} \cdot \cancel{e^C} = C_1 e^{x^5/5}$$

$$y-2 = C_1 e^{x^5/5}$$

$$\boxed{y = C_1 e^{x^5/5} + 2}$$ ← general soln → $\boxed{f(x) = C_1 e^{x^5/5} + 2}$

16D $f(0) = 0$

$$y(0) = C_1 e^0 + 2 = 0$$
$$C_1 + 2 = 0$$
$$C_1 = -2$$

$$\boxed{y = f(x) = -2e^{x^5/5} + 2}$$

CALCULATORS ARE PERMITTED—

But all CALCULUS work (integrating, deriving) must be shown by hand. You may use your calculator for simple arithmetic with large quantities and to check your work.

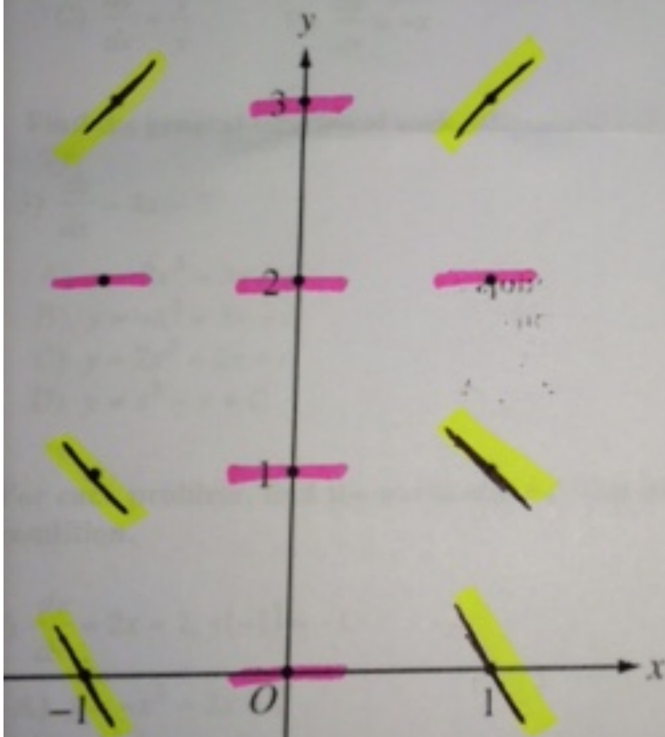
This review is identical in format to the exam. Only the actual values in the questions will vary.

Multiple-Choice Answers: (#1-15) You may write on the actual exam, but only answers recorded HERE will count towards your grade. Please write in capital letters to clearly distinguish between A and D. *(3 points each.)*

Free-Response: (Total 25 points.)
You must show a reasonable amount of work that leads to your answer. Where it is impossible to show work, explain the mental leaps that you made to draw your conclusion.

16. Consider the differential equation $\frac{dy}{dx} = x^4(y-2)$.

A. On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. **Make a table to find the slopes and SHOW ALL WORK.** *(14 points)*



x	y	$m=x^4(y-2)$
−1	0	1(−2) = −2
−1	1	1(−1) = −1
−1	2	1(0) = 0
−1	3	1(1) = 1
0	0	0(−2) = 0
0	1	0(−1) = 0
0	2	0(1) = 0
0	3	0(2) = 0
1	0	1(−2) = −2
1	1	1(−1) = −1
1	2	1(0) = 0
1	3	1(1) = 1

While the slope field in part (a) is drawn at only twelve points, it is defined at every point in
ie. Describe all points in the xy-plane for which the slopes are negative. SHOW ALL WORK.

ind the general solution, y = f(x), to the differential equation. SHOW ALL WORK. *(5 points)*

ind the particular solution, y = f(x), to the differential equation with the initial condition f(
W ALL WORK. *(3 points.)*