CHAB Final Review Packet

This review is a full AP diagnostic exam. Although there are 45 multiple-choice questions and 6 free-response questions on this review, your actual final exam will be much shorter.

FINAL EXAM FORMAT:

- 20 multiple-choice questions.
- 1 free-response question (just like those warm-up quizzes you did early in the year.)
- Calculator permitted for ALL question.

FINAL EXAM GRADING:

CALCULATORS ARE PERMITTED ON THIS EXAM.

Scoring: These are AP level questions. This exam will be curved.

RAW SCORE	LETTER GRADE	GRADEBOOK GRADE
20 to 24	A+	100%
15 to 19	A	90%
10 to 14	B	80%
5 to 9	C	70&
1 to 4	F	60%
0	F	0%

MUTLIPLE-CHOICE TOPICS ON THE FINAL EXAM:

- Indefinite Integration: 4 problems
- Limits
- Tangent Line Equations/Equation of a Line: 2 problems
- Particle Motion
- Differential Equations
- Trapezoidal Area Approximation
- Calculus with Piecewise Functions
- Derivatives of Exponential Functions
- Extrema, Increasing/Decreasing
- Implicit Differentiation
- Integrals give a graph
- Points of Inflection
- Volume of Solids in Revolution
- Volume of Solids with Defined Cross Sections
- Slope Fields
- Related Rates

AB PRACTICE TEST 2
Section I, Part A: Multiple-Choice Questions
Time: 55 minutes
Number of Questions: 28

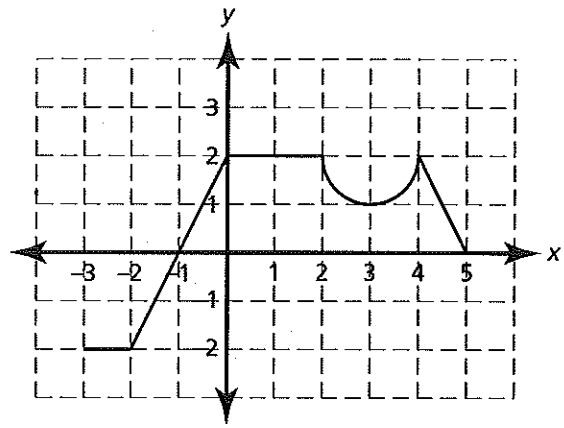
A calculator may not be used on this part of the examination.

1. In what interval is $f(x) = \ln(x^2 - 1)$ decreasing?
   (A) $|x| > 1$
   (B) $|x| \geq 1$
   (C) $x < -1$
   (D) $x > 1$
   (E) $x > 0$

2. Find the limit $\lim\limits_{x \to 0} \dfrac{\cos x}{|x|}$.
   (A) 0
   (B) 1
   (C) −1
   (D) π
   (E) The limit does not exist.

3. $\displaystyle\int \left[3^{-x} + \frac{1}{x}\right] dx = ?$
   (A) $3^{-x} + \ln|x| + C$
   (B) $-3^{-x} - \dfrac{1}{x^2} + C$
   (C) $-3^{-x}\ln 3 + \ln|x| + C$
   (D) $-\dfrac{3^{-x}}{\ln 3} + \ln|x| + C$
   (E) $-\dfrac{3^{-x}}{\ln 3} - \dfrac{1}{x^2} + C$

4. Find $\dfrac{dy}{dx}$ for $\cos(x+y) = x$.
   (A) $-\csc(x+y) - 1$
   (B) $\dfrac{\cos(x+y)}{\sin^2(x+y)}$
   (C) $\dfrac{x}{1-x^2}$
   (D) $-\sin(x+y) \cdot \cot(x+y)$
   (E) $-\sin(x+y) - 1$



The graph of $f(x)$ consists of four line segments and a semicircle as shown above in the closed interval $-3 \leq x \leq 5$. Let g be the function given by $g(x) = \int_0^x f(t)\, dt$. Use this information for problems 5–7.

5. What is $g(-1) + g'(-1) + g''(-1)$?
   (A) −1
   (B) 0
   (C) 1
   (D) 2
   (E) 3

6. What is $\int_{-3}^{5} f(t)\, dt$?
   (A) $7 - \pi$
   (B) $7 - \dfrac{\pi}{2}$
   (C) $7 - \dfrac{\pi}{4}$
   (D) $12 - \dfrac{\pi}{2}$
   (E) $12 - \dfrac{\pi}{4}$

7. Which of the following statements is false for $g(x)$?
   (A) The absolute maximum for $g(x)$ occurs at $x = 5$.
   (B) A relative minimum for $g(x)$ occurs at $x = -1$.
   (C) A point of inflection for $g(x)$ occurs at $x = 3$.
   (D) $g(x)$ has roots at $x = 0$ and $x = -2$.
   (E) $g(x)$ is concave down in the open interval $-2 < x < -1$.

8. The position of a particle moving along the x-axis is given by $x(t) = 2 + 3t - t^3$. What is the speed of the particle at $t = 4$?
   (A) -50
   (B) -45
   (C) 32
   (D) 45
   (E) 50

9. A region R is bounded by the curve $x = y^2 - 1$ and the y-axis. What is the volume generated when region R is rotated about the y-axis?
   (A) $\frac{\pi}{2}$
   (B) $\frac{8\pi}{15}$
   (C) π
   (D) $\frac{16\pi}{15}$
   (E) $\frac{4\pi}{3}$

10. Which of the following statements is true for $f(x) = \sqrt[3]{x} + 1$?

    I. $f(x)$ is always increasing, $x \neq 0$.
    II. The tangent to the curve at $x = 0$ is horizontal.
    III. The Mean Value Theorem can be applied to $f(x)$ in the closed interval $-1 \leq x \leq 1$.

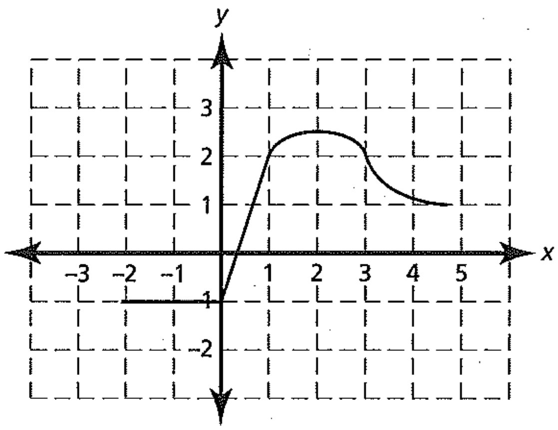
    (A) I only
    (B) II only
    (C) III only
    (D) II and III only
    (E) I, II, and III

11. The acceleration of a model car along an incline is given by $a(t) = \frac{t^2 + t}{t^2 + 1}\text{cm/sec}^2$, for $0 \leq t < 1$. If $v(0) = 1$ cm/sec, what is $v(t)$?
    (A) $\tan^{-1} t + \frac{1}{2}\ln(t^2 + 1) + 1$ cm/sec
    (B) $\tan^{-1} t - \frac{1}{2}\ln(t^2 + 1) + 1$ cm/sec
    (C) $t - \frac{1}{2}\ln(t^2 + 1) - \tan^{-1} t + 1$ cm/sec
    (D) $t + \frac{1}{2}\ln(t^2 + 1) + \tan^{-1} t + 1$ cm/sec
    (E) $t + \frac{1}{2}\ln(t^2 + 1) + \tan^{-1} t + 1$ cm/sec

t	$R(t)$
0	12
2	18
4	10
6	15
8	12
10	16
12	8

12. Water is dripping into a vase at a variable rate. The rate, $R(t)$ in cm^3/min, is recorded every 2 mins for 12 mins, as listed in the chart above. Using a right Riemann sum with 3 equal intervals, find the approximate average rate at which the water drips into the vase over the 12 mins.
    (A) $10\ \text{cm}^3/\text{min}$
    (B) $10\frac{1}{3}\ \text{cm}^3/\text{min}$
    (C) $16\frac{1}{3}\ \text{cm}^3/\text{min}$
    (D) $40\ \text{cm}^3/\text{min}$
    (E) $41\frac{1}{3}\ \text{cm}^3/\text{min}$

13. If $h''(x) = e^{x-1}(2x-1)^2(x-3)^3(4x+5)$, then $h(x)$ has how many points of inflection?
(A) 4
(B) 3
(C) 2
(D) 1
(E) 0



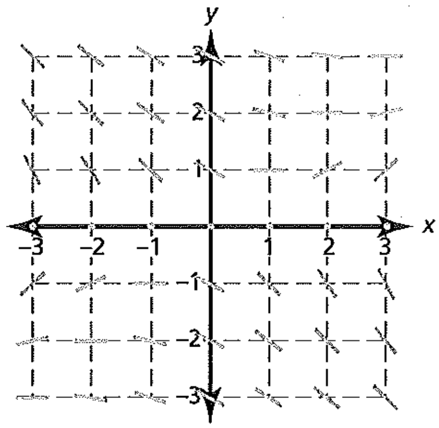
14. The graph of $f'(x)$ is shown in the figure above. If $\int_1^2 f'(x)\,dx = k$, what is $f(2)-f(-1)$?
(A) $k+\frac{11}{6}$
(B) $k+\frac{3}{2}$
(C) $k-\frac{1}{2}$
(D) $k-1$
(E) $k-\frac{4}{3}$

15. $\int \frac{1}{16+x^2}\,dx =$
(A) $4\tan^{-1}x+C$
(B) $\frac{1}{4}\tan^{-1}x+C$
(C) $\tan^{-1}\frac{x}{4}+C$
(D) $\frac{1}{4}\tan^{-1}4x+C$
(E) $\frac{1}{4}\tan^{-1}\frac{x}{4}+C$

16. A particle moves along the x-axis so that its velocity for $t \geq 0$ is given by $v(t) = -t^3+\frac{9}{4}t^4$. At what time is the acceleration of the particle a minimum?
(A) $\frac{2}{9}$
(B) $\frac{1}{3}$
(C) $\frac{4}{9}$
(D) 0
(E) none of these

17. What is the value of $\lim_{h\to 0}\frac{\sin^{-1}(1+h)-\frac{\pi}{2}}{h}$?
(A) 1
(B) 0
(C) −1
(D) $\frac{\pi}{2}$
(E) The limit does not exist.

18. Which of the following is the equation of the tangent to the curve of $f(x)=e^{\sin x}+x$ at $x=0$?
(A) $y=2x-1$
(B) $y=2x+1$
(C) $y=2x$
(D) $y=1$
(E) $y=0$



19. Which of the following statements matches the slope field shown above?
(A) $\frac{dy}{dx}=\frac{x-y}{2y}$
(B) $\frac{dy}{dx}=\frac{x+y}{2y}$
(C) $\frac{dy}{dx}=\frac{x-y}{2x}$
(D) $\frac{dy}{dx}=\frac{x+y}{2x}$
(E) $\frac{dy}{dx}=\frac{2y}{x-y}$

20. If $F(x)=\int_{3x-2}^{2} f(2t)\,dt$, what is $F'(x)$ in terms of x?
(A) $3f(6x-4)$
(B) $f(3x-2)$
(C) $\frac{1}{2}f(3x-2)$
(D) $-f(3x-2)$
(E) $-3f(6x-4)$

21. What is the slope of the line normal to the curve $h(x)=\sqrt{5x^3-2x+1}$ at the point where $x=1$?
(A) $-\frac{13}{4}$
(B) $-\frac{4}{13}$
(C) $\frac{4}{13}$
(D) $\frac{13}{4}$
(E) none of these

22. Let $y=2x(\sin 2x+x\cos 2x)$ in the interval $0\le x\le\frac{\pi}{2}$. What is the average rate of change of y with respect to x in this interval?
(A) $-\pi$
(B) $-\frac{\pi}{2}$
(C) 0
(D) $\frac{\pi}{2}$
(E) π

23. Find k so that the relative minimum of $f(x)=x^3-\ln(k+x)$ occurs at $x=1$.
(A) 2
(B) $\sqrt{3}$
(C) $\frac{1}{3}$
(D) 0
(E) $-\frac{2}{3}$

24. The concentration of an anti-inflammatory drug in the bloodstream t mins after taking a single dose is $C(t)=\frac{2t}{8100+t^2}$, $t\ge 0$. At what time is the concentration the greatest?
(A) 90 minutes
(B) $30\sqrt{6}$ minutes
(C) $30\sqrt{3}$ minutes
(D) $15\sqrt{6}$ minutes
(E) none of these

25. If $\lim_{x\to\infty} \frac{ae^x}{x+e^x} = 3$, what is a?
(A) 9
(B) 6
(C) 3
(D) none of these
(E) No such value of a exists.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	4	$\frac{2}{3}$	$-\frac{5}{2}$
2	4	2	$\frac{4}{3}$	$-\frac{3}{2}$
4	8	1	$\frac{8}{3}$	$-\frac{1}{2}$

26. Use the values listed in the chart above to find the value of
$\frac{d}{dx}\left[f\left(g(x^2)\right)\right]$ when $x = 2$.
(A) -8
(B) -4
(C) $-\frac{4}{3}$
(D) $\frac{2}{3}$
(E) $\frac{8}{3}$

27. The volume of an open rectangular box is 8 cm^3, and the length of the rectangular base is twice as long as its width. What is the width of the base so that the surface area of the open box is minimized?
(A) $\sqrt[3]{3}$
(B) $\sqrt[3]{6}$
(C) $\sqrt{3}$
(D) 2
(E) $\sqrt{6}$

28. The base of a solid is bounded by the curve $y = \sqrt{x+1}$, the x-axis, and the line $x = 1$. The cross sections, taken perpendicular to the x-axis, are squares. Find the volume of the solid.
(A) $\frac{1}{2}$
(B) 1
(C) $\frac{4\sqrt{2}}{3}$
(D) 2
(E) $\frac{8}{3}$

Section I, Part B: Multiple-Choice Questions
Time: 50 minutes
Number of Questions: 17

A calculator may be used on this part of the examination.

29. Suppose $g(0) = 4$, $g'(0) = 8$, and $g''(0) = -12$. If $h(x) = \sqrt{g(x)}$, what is $h''(0)$?
(A) -5
(B) $-\frac{13}{4}$
(C) $-\frac{1}{32}$
(D) $\frac{3}{8}$
(E) 1

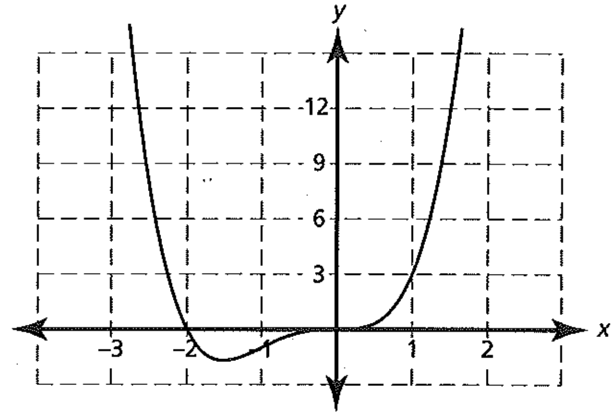
30. If $g''(x) = \frac{x}{2+e^x}$ and $g'(0) = -1$, what is $g'(3)$?
(A) -0.864
(B) -0.473
(C) 0.136
(D) 0.527
(E) 1.527

31. Two cars are converging on a point P as they drive at right angles to each other. Car A is traveling at 60 miles per hour and car B is traveling at 50 miles per hour. At the instant when car A is 12 miles from the point P and car B is 10 miles from the point P, at what rate is the distance between the cars decreasing?
(A) 55 miles per hour
(B) 55.455 miles per hour
(C) 76.882 miles per hour
(D) 78.102 miles per hour
(E) none of these

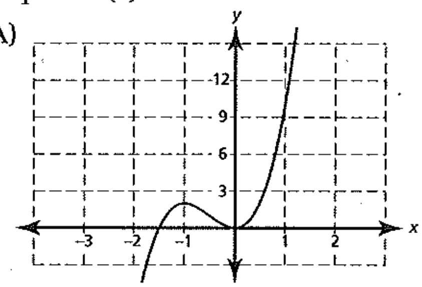
32. A line tangent to the curve $f(x) = \frac{1}{2^{2x}}$ at the point $(a, f(a))$ has a slope of -1. What is the x-intercept of this tangent?
(A) 0.236
(B) 0.500
(C) 0.721
(D) 0.957
(E) 1.000

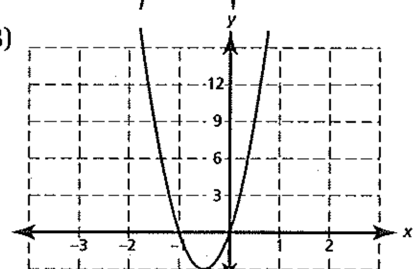
33. Consider the function $g(x) = \tan(x+2)$ in the open interval $-4 < x < 5$. How many times are the tangents to $g(x)$ parallel to the line $y = 2x - 1$?
(A) never
(B) 2
(C) 4
(D) 5
(E) an infinite number of times

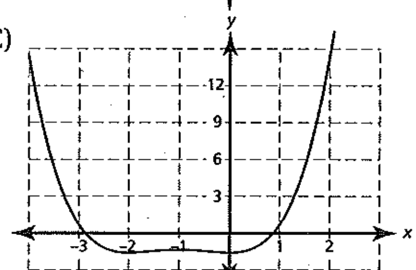
34. What is the sum of all k values that satisfy $\int_1^{2k} x - \frac{k}{x^2}\, dx = 15$?
(A) 3
(B) $\frac{1}{2}$
(C) 1
(D) 0
(E) $-\frac{1}{2}$

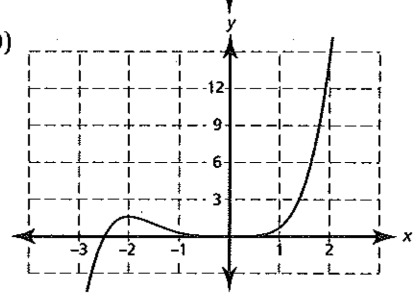


35. The graph of $f'(x)$ is shown above. Which of the following could be a graph of $f(x)$?

(A)


(B)


(C)


(D)


(E) none of these

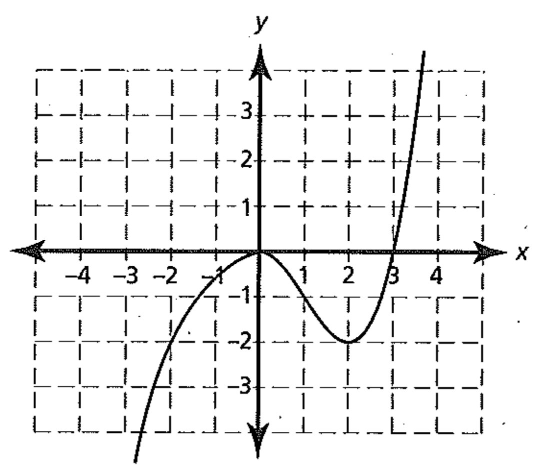
t	0	1	2	3	4
$H(t)$	0	1.3	1.5	2.1	2.6

36. A small plant is purchased from a nursery and the change in the height of the plant is measured at the end of every day for four days. The change in the height of the plant is listed in the chart above where $H(t)$ is in inches per day and t is in days. Using the Trapezoidal Rule, which of the following represents an estimate of the average rate of growth of the plant during the 4-day period?
   (A) $\frac{1}{4}(0+1.3+1.5+2.1+2.6)$
   (B) $\frac{1}{4}\left[\frac{1}{2}(0+1.3+1.5+2.1+2.6)\right]$
   (C) $\frac{1}{4}\left\{\frac{1}{2}[0+2(1.3)+2(1.5)+2(2.1)+2.6]\right\}$
   (D) $\frac{1}{4}\left\{\frac{1}{2}[0+2(1.3)+2(1.5)+2(2.1)+2(2.6)]\right\}$
   (E) $\frac{1}{4}\left\{\frac{1}{4}[0+2(1.3)+2(1.5)+2(2.1)+2.6]\right\}$

37. A bug travels along the y-axis such that its velocity, in centimeters per second, is given by $v(t)=(t^2-1)e^{t^2+2t}$, and t is in seconds. How far does the bug travel in the first 1.5 sec?
   (A) 2.257 cm
   (B) 3.386 cm
   (C) 27.155 cm
   (D) 29.412 cm
   (E) 31.669 cm

38. Consider a differentiable function $f(x)$ that has an x-intercept of 2 and a y-intercept of 4. In the interval $0 < x < 2$, $f(x)$ is decreasing and concave down. Which of the following must be true?
   I. The Mean Value Theorem is satisfied when $c = 1$.
   II. $\int_0^1 f(x)\,dx > \int_1^2 f(x)\,dx$
   III. A tangent approximation of the function for any value in the interval $0 < x < 2$ will underestimate the function value.

   (A) I only
   (B) II only
   (C) III only
   (D) I and II only
   (E) II and III only



Let $f(x) = x^2 + \int_{-2}^{x} g(t)\,dt$, where $g(x)$ is shown in the graph above. Use this graph to answer problems 39–41.

39. What is $f(-2)$?
   (A) –6
   (B) –4
   (C) 0
   (D) 2
   (E) 4

40. What is $f'(-2)$?
   (A) –6
   (B) –4
   (C) 0
   (D) 2
   (E) 4

41. What is $f''(2)$?
    (A) –6
    (B) –4
    (C) 0
    (D) 2
    (E) 4

42. What is the area of the regions between $f(x) = 2x\sin(2x)$ and $g(x) = -2x\cos 2x$ on the interval $0 \le x \le \frac{\pi}{2}$?
    (A) 0.571
    (B) 0.595
    (C) 1.166
    (D) 1.178
    (E) 1.761

43. The graph of $f'(x)$ is given above, in the interval $-8 \le x \le 5$. In which interval(s) is $f(x)$ decreasing and concave down?
    (A) $-2 < x < 0$
    (B) $-2 < x < 3$ and $3 < x < 5$
    (C) $-2 < x < 0$ and $3 < x < 5$
    (D) $-6 < x < 0$ and $3 < x < 5$
    (E) $-8 < x < -3$ and $2 < x < 5$

44. The rate of decay of a radioactive isotope is directly proportional to the amount remaining. If the half-life of the radioactive isotope, Einsteinium, is 276 days and a sample initially weighs 25 grams, what is its rate of decay on the 120th day?
    (A) –0.046 grams per day
    (B) –0.031 grams per day
    (C) –0.003 grams per day
    (D) –0.002 grams per day
    (E) –0.001 grams per day

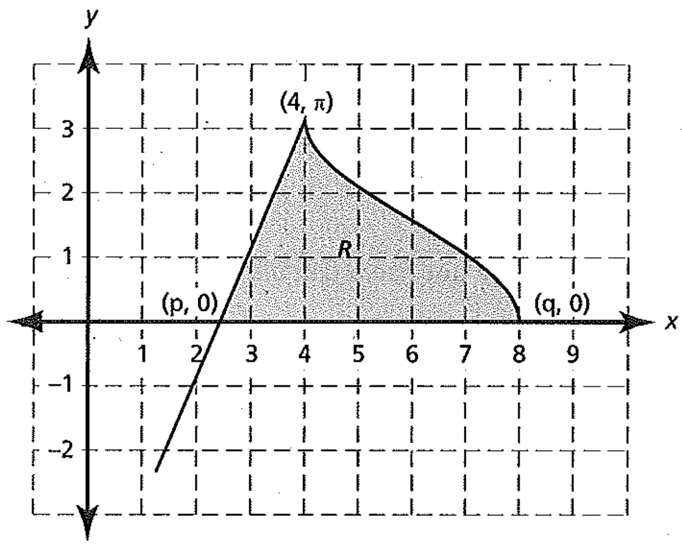
45. A region is bounded by the function $y = \sin^{-1}(x-1) + \frac{\pi}{2}$, the line $x = 2$, and the x-axis. A solid is formed when the region is rotated about the line $x = 2$. What is the radius of the volume of rotation?
    (A) $r = \sin^{-1}(x-1) + \frac{\pi}{2}$
    (B) $r = 2 - \left(\sin^{-1}(x-1) + \frac{\pi}{2}\right)$
    (C) $r = 2 - \sin\left(y - \frac{\pi}{2}\right)$
    (D) $r = 1 - \sin\left(y - \frac{\pi}{2}\right)$
    (E) $r = 1 + \sin\left(y - \frac{\pi}{2}\right)$

Section II
Free-Response Questions
Time: 1 hour and 30 minutes
Number of Problems: 6

Part A
Time: 45 minutes
Number of Problems: 3

You may use a calculator for any problem in this section.

1. Region R is bounded by the functions $f(x) = 2(x-4)+\pi$, $g(x) = \cos^{-1}\left(\frac{x}{2}-3\right)$, and the x-axis as shown in the figure to the right.
   a. What is the area of region R?
   b. Find the volume of the solid generated when region R is rotated about the x-axis.
   c. Find all values c for $f(x)$ and $g(x)$ in the closed interval $p \le c \le q$ for which each function equals the average value in the indicated interval.



2. A particle is moving along the x-axis with velocity $v(t) = \ln(x+3) - e^{\frac{x}{2}-1}(\cos x)$, for $0 \le t \le 8$. The initial position of the particle is -1.6.
   a. At what time(s) in the open interval $0 < t < 8$ does the particle change direction? Justify your answer.
   b. Where is the particle when it is farthest to the left?
   c. How far does the particle travel in the interval $3 \le t \le 6$?
   d. At what times in the closed interval, $0 \le t \le 8$, is the speed of the particle decreasing? Justify your answer.

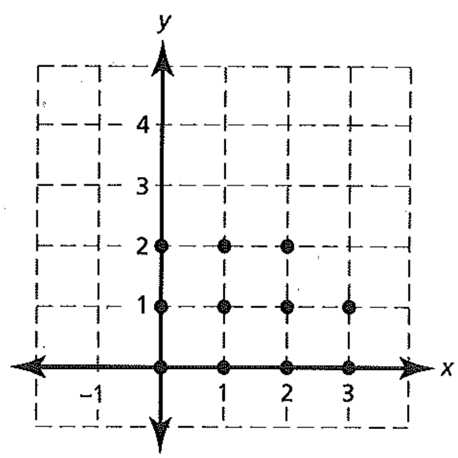
3. A large flea market is held at the local fairgrounds on the first Saturday of each month. The rates at which people enter and leave the fairgrounds are recorded for a 3-hour period beginning when the market is open to the public. The rate at which people arrive is modeled by the function $A(t) = 45\sin(0.03t - 0.7) + 71$. The function $L(t) = 42\sin(0.034t - 1.52) + 42$ models the rate at which people leave the fairgrounds. Both $A(t)$ and $L(t)$ are measured in people per minute and t is measured for $0 \le t \le 180$ minutes. When the count begins at $t = 0$, there are already 1572 people in the flea market area of the fairgrounds.
   a. How many additional people arrive for the flea market during the 3-hour period after it opens to the public?
   b. Write an expression for $P(t)$, the total number of people at the flea market at time t.
   c. Find the value of $P'(75)$ and explain its meaning.
   d. For $0 \le t \le 180$, at what time is the rate of change of people at the flea market at a maximum? What is the maximum rate of change? Justify your answers.

Part B
Time: 45 minutes
Number of Problems: 3

You may not use a calculator for any problem in this section.

During the timed portion for Section II, Part B, you may continue to work on the problems in Part A without the use of a calculator.

4. Consider the differential equation
   $\dfrac{dy}{dx} = (y+1)(1-x)$ for $y > -1$.
   a. On the axes provided, sketch a slope field for the given differential equation at the 11 points indicated.
   b. If $y(0) = 1$, then find the particular solution $y(x)$ to the given differential equation.
   c. Draw a function through the point (0, 1) on your slope field which represents an approximate solution of the given differential equation with initial condition $y(0) = 1$.

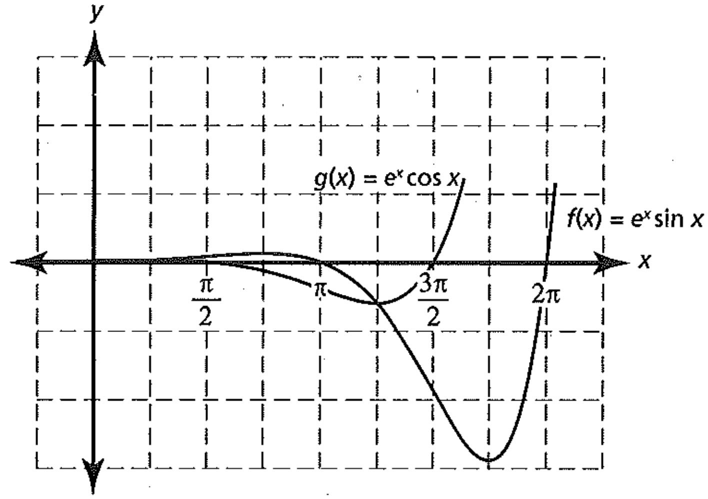


5. Consider the function $h(x) = 3x^2 - \sqrt{x+1}$.
   a. Evaluate $\dfrac{1}{3-(-1)}\int_{-1}^{3}\left(3x^2 - \sqrt{x+1}\right)dx$ and interpret its meaning.
   b. What is the equation of the tangent to $h(x)$ at $x = 0$?
   c. Use the tangent found in part b to approximate $h(x)$ at $x = -0.01$.
   d. Is the approximation, found in part c, greater or less than the actual value of $h(x)$ at $x = -0.01$? Justify your answer using calculus.

6. Consider the functions $f(x) = e^x \sin x$ and $g(x) = e^x \cos x$, as shown in the sketch to the right, in the closed interval $0 \le x \le 2\pi$.

   a. Let $D(x) = f(x) - g(x)$ be the vertical distance between the functions. Find the value of x in the open interval $\frac{\pi}{4} < x < \frac{5\pi}{4}$, where $D(x)$ is a maximum value. Explain your reasoning.

   b. At what value of x in the open interval $\frac{\pi}{4} < x < \frac{5\pi}{4}$ is the rate of change of $D(x)$ increasing the most rapidly? Explain your reasoning.



   c. For $H(x) = \frac{e^x}{\sin x} + g(x)$, find the x-coordinate of all points at which $H(x)$ has horizontal tangents on the open interval $0 < x < 2\pi$.