Answers and Answer Explanations

Using the table below, score your test. Determine how many questions you answered correctly and how many you answered incorrectly. Additional information about scoring is at the end of the Practice Test.

1. C	2. E	3. D	4. A	5. C
6. B	7. E	8. D	9. D	10. A
11. E	12. A	13. C	14. C	15. E
16. A	17. E	18. B	19. A	20. E
21. B	22. A	23. E	24. A	25. C
26. C	27. B	28. D	29. A	30. B
31. D	32. D	33. D	34. B	35. D
36. C	37. E	38. B	39. E	40. A
41. D	42. E	43. C	44. A	45. D

MULTIPLE-CHOICE QUESTIONS

1. ANSWER: (C) The domain for $f(x) = \ln(x^2 - 1)$ is $|x| > 1$. With $f'(x) = \dfrac{2x}{x^2-1} = 0 \Rightarrow x = 0$, but $x = 0$ is not in the domain of $f(x)$. The behavior of the curve is analyzed using the sign chart below.

x	$-\infty < x < -1$	$1 < x < \infty$
$f'(x)$	negative	positive
$f(x)$	decreasing	increasing

Therefore the curve is decreasing when $x < -1$.
(*Calculus* 7th ed. pages 174–180 / 8th ed. pages 179–185)

2. ANSWER: (E) $\lim\limits_{x\to 0} \dfrac{\cos x}{|x|} = \dfrac{1}{0}$, which does not exist. (The value of the limit from both the left and right side of 0 is ∞.)
(*Calculus* 7th ed. pages 80–84 / 8th ed. pages 83–87)

3. ANSWER: (D) By substituting $u = -x$ and $du = -dx$,
$$\int 3^{-x} + \frac{1}{x}dx = -\int 3^u\,du + \int \frac{1}{x}dx = -\frac{3^{-x}}{\ln 3} + \ln|x| + C$$
(*Calculus* 7th ed. pages 351–356, 324–330 / 8th ed. pages 360–365, 332–337)

4. ANSWER: (A) $\frac{d}{dx}\left[\cos(x+y)\right]=\frac{d}{dx}(x) \Rightarrow -\sin(x+y)\cdot\left(1+\frac{dy}{dx}\right)=1$

$$1+\frac{dy}{dx}=\frac{1}{-\sin(x+y)}=-\csc(x+y)$$

$$\frac{dy}{dx}=-\csc(x+y)-1$$

(*Calculus* 7th ed. pages 137–143 / 8th ed. pages 141–148)

5. ANSWER: (C) $g(-1)=\int_0^{-1} f(t)\ dt=-\int_{-1}^{0} f(t)\ dt=-\frac{1}{2}(1)(2)=-1$

$g'(x)=f(x)\Rightarrow g'(-1)=f(-1)=0$

$g''(x)=f'(x)\Rightarrow g''(-1)=f'(-1)=2$

$g(-1)+g'(-1)+g''(-1)=-1+0+2=1$

(*Calculus* 7th ed. pages 275–287 / 8th ed. pages 282–294)

6. ANSWER: (B) Solve the integral by adding up the geometric areas pictured.

$$\int_{-3}^{5} f(t)\ dt=1(-2)+\frac{1}{2}(1)(-2)+\frac{1}{2}(1)(2)+2(2)+\left(2(2)-\frac{\pi}{2}\right)+\frac{1}{2}(1)(2)$$

$$=-2-1+1+4+4-\frac{\pi}{2}+1$$

$$=7-\frac{\pi}{2}$$

(*Calculus* 7th ed. pages 265–274 / 8th ed. pages 271–281)

7. ANSWER: (E) The function $g(x)$ is increasing for $x>0$, so the absolute maximum occurs at $x=5$. The relative minimum occurs where $g'(x)$ changes from negative to positive, so the relative minimum occurs at $x=-1$. A point of inflection occurs where the slope of $f(x)$, $f'(x)=g''(x)$, changes from negative to positive, so a point of inflection occurs where $x=3$. The roots of $g(x)$ occur where $\int_0^x f(t)\ dt=0$, and this occurs when $x=0$ or $x=-2$. Therefore the false statement is E, because $g''(x)=f'(x)$ is greater than zero in the open interval $-2<x<-1$, which indicates that the curve is concave up.

(*Calculus* 7th ed. pages 275–287 / 8th ed. pages 282–294)

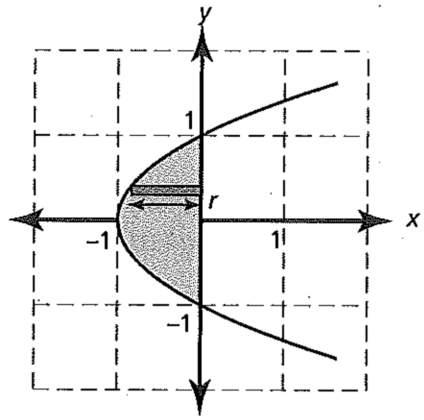
8. ANSWER: (D) Speed $=|v(t)|=|x'(t)|$. Since $x'(t)=3-3t^2$, then $|v(4)|=\left|3-3(4)^2\right|=45$

(*Calculus* 7th ed. pages 105–116 / 8th ed. pages 107–118)

9. ANSWER: (D) Using the disk method and integrating with respect to y:
$V = \pi\int_{-1}^{1} x^2\,dx = 2\pi\int_{0}^{1}(y^2-1)^2\,dy$.

$$V = 2\pi\int_{0}^{1}(y^4-2y^2+1)dy$$
$$= 2\pi\left(\frac{1}{5}y^5-\frac{2}{3}y^3+y\right)\Big|_0^1$$
$$= 2\pi\left(\frac{1}{5}-\frac{2}{3}+1\right)=\frac{16\pi}{15}$$

(*Calculus* 7th ed. pages 421–431 / 8th ed. pages 456–466)



10. ANSWER: (A) Case I: $f'(x)=\frac{1}{3x^{\frac{2}{3}}}$ and is not defined at $x=0$.
Therefore, $f'(x)$ is always greater than zero, except at $x=0$. (Therefore, the statement is true)

Case II: The function is not differentiable at $x=0$ and since the slope of the tangent line is undefined at $x=0$, the tangent line is vertical. (So, the statement is false.)

Case III: The Mean Value Theorem is not applicable in the stated interval, because the function is not differentiable at $x=0$. (Thus, the statement is false.)
(*Calculus* 7th ed. pages 160–191 / 8th ed. pages 164–197)

11. ANSWER: (E) Rewrite $a(t)$ using long division:
$$\frac{t^2+t}{t^2+1}\Rightarrow 1+\frac{-1+t}{t^2+1}\Rightarrow 1+\frac{1}{t^2+1}+\frac{t}{t^2+1}$$
$$v(t)=\int a(t)dt=\int\left(1+\frac{-1}{t^2+1}+\frac{t}{t^2+1}\right)dt=t-\tan^{-1}t+\frac{1}{2}\ln(t^2+1)+C.$$

If $v(0)=1$, then $C=1$ and finally, $v(t)=t-\tan^{-1}t+\frac{1}{2}\ln(t^2+1)+1$.
(*Calculus* 7th ed. pages 324–330, 388–394 / 8th ed. pages 332–337, 380–387)

12. ANSWER: (A) For $n=3$ the width of each rectangle is 4. Since $f_{avg}=\frac{1}{b-a}\int_a^b f(x)dx$ and $\int_a^b f(x)dx$ is approximated by using a right Riemann sum, then
$$R_{avg}=\frac{1}{12}\int_0^{12}R(t)dt\approx\frac{1}{12}(4)(10+12+8)=10\text{ cm}^3/\text{min}$$
(*Calculus* 7th ed. pages 265–274 / 8th ed. pages 271–281)

13. ANSWER: (C) $h(x)$ has a point of inflection where $h''(x)$ changes sign. For $h''(x)=e^{x-1}(2x-1)^2(x-3)^3(4x+5)$, the factor e^{x-1} never equals zero.

x	$-\infty < x < -\frac{5}{4}$	$-\frac{5}{4} < x < \frac{1}{2}$	$\frac{1}{2} < x < 3$	$3 < x < \infty$
$h''(x)$	positive	negative	negative	positive

Therefore, there are two points of inflection, one at $x = -\frac{5}{4}$ and the other at $x = 3$, where $h''(x)$ changes from positive to negative or negative to positive.
(*Calculus* 7th ed. pages 184–191 / 8th ed. pages 190–197)

14. ANSWER: (C) Since $\int_a^b f'(x)\,dx = f(b)-f(a)$, then

$f(2)-f(-1)=\int_{-1}^{2} f'(x)\,dx$. Then $\int_{-1}^{2} f'(x)\,dx \Rightarrow$

$\int_{-1}^{1} f'(x)\,dx + \int_{1}^{2} f'(x)\,dx \Rightarrow \int_{-1}^{1} f'(x)\,dx + k$. Thus,

$$f(2)-f(-1)=\left[1(-1)+\frac{1}{2}\left(\frac{1}{3}\right)(-1)+\frac{1}{2}\left(\frac{2}{3}\right)(2)\right]+k$$
$$=\left[-1-\frac{1}{6}+\frac{2}{3}\right]+k$$
$$=k-\frac{1}{2}$$

(*Calculus* 7th ed. pages 265–287 / 8th ed. pages 271–294)

15. ANSWER: (E) Since $\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\tan^{-1}\frac{u}{a}+C$,

then $\int \frac{1}{16+x^2}\,dx = \frac{1}{4}\tan^{-1}\frac{x}{4}+C$
(*Calculus* 7th ed. pages 388–394 / 8th ed. pages 380–387)

16. ANSWER: (A) Acceleration has a relative maximum or minimum when $a'(t)=0$. So, $v(t)=-t^3+\frac{9}{4}t^4 \Rightarrow a(t)=-3t^2+9t^3$ and finally,

$a'(t)=-6t+27t^2=0 \Rightarrow -3t(2-9t) \Rightarrow t=\frac{2}{9}$ or 0

t	$0 < t < \frac{2}{9}$	$\frac{2}{9} < t < \infty$
$a'(t)$	negative	positive
$a(t)$	decreasing	increasing

Therefore $a(t)$ is at a minimum when $t = \frac{2}{9}$, where $a(t)$ changes from decreasing to increasing.
(*Calculus* 7th ed. pages 184–191 / 8th ed. pages 190–197)

17. ANSWER: (E) $\lim_{h\to 0}\frac{\sin^{-1}(1+h)-\frac{\pi}{2}}{h}$ is in the form of the definition of the derivative, $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=f'(x)$, with the function, $f(x)=\sin^{-1}(x)$ and $x=1$, because $\sin^{-1}(1)=\frac{\pi}{2}$. Then, $f'(x)=\frac{1}{\sqrt{1-x^2}}$ and $f'(1)=\frac{1}{\sqrt{1-1^2}}$ does not exist.
(*Calculus* 7th ed. pages 380–387, 94–104 / 8th ed. pages 360–365, 96–106)

18. ANSWER: (B) For $f(x)=e^{\sin x}+x$, $f(0)=e^0+0=1$ and $f'(x)=e^{\sin x}\cos x+1$, with $f'(0)=e^0\cos 0+1=2$. Since $f(0)=1$ and $f'(0)=m=2$, then the equation of the tangent is $y-1=2(x-0)\Rightarrow y=2x+1$.
(*Calculus* 7th ed. pages 341–350 / 8th ed. pages 350–359)

19. ANSWER: (A) Observe that the segments with slopes of zero occur where x and y are equal and that the segments with vertical slopes occur where $y=0$. Thus the statement that matches the slope field is $\frac{dy}{dx}=\frac{x-y}{2y}$.
(*Calculus* 7th ed. pages A2–A3 / 8th ed. pages 404–408)

20. ANSWER: (E) The Second Fundamental Theorem states that if $F(x)=\int_a^{g(x)} f(u)\,du$, then $F'(x)=f[g(x)]\cdot g'(x)$. First, rewrite $F(x)=\int_{3x-2}^{2} f(2t)\,dt \Rightarrow F(x)=-\int_2^{3x-2} f(2t)\,dt$, where $g(x)=3x-2$. By substitution of $u=2t$ and $du=2\,dt$, change the limits of the integral (where $t=2$, then $u=4$, and where $t=3x-2$, $u=6x-4$) and the integral becomes $F(x)=-\frac{1}{2}\int_4^{6x-4} f(u)du$. Finally,

$F'(x)=-\frac{1}{2}[f(6x-4)]\cdot 6=-3f(6x-4)$
(*Calculus* 7th ed. pages 275–287 / 8th ed. pages 282–294)

21. ANSWER: (B) With $h'(x)=\frac{15x^2-2}{2\sqrt{5x^3-2x+1}}$, $h'(1)=\frac{15-2}{2\sqrt{5-2+1}}=\frac{13}{4}$. A normal line is perpendicular to the tangent at the point of tangency, so $m_{\text{normal}}=-\frac{4}{13}$.
(*Calculus* 7th ed. pages 127–136 / 8th ed. pages 130–140)

22. ANSWER: (A) The average rate of change of a function in a closed interval is given by $\frac{f(b)-f(a)}{b-a}$. Then the average rate of change of

y in the closed interval $0 \le x \le \frac{\pi}{2}$ is

$$\frac{y\left(\frac{\pi}{2}\right)-y(0)}{\left(\frac{\pi}{2}\right)-0}=\frac{2\left(\frac{\pi}{2}\right)\left[\sin 2\left(\frac{\pi}{2}\right)+\frac{\pi}{2}\cos 2\left(\frac{\pi}{2}\right)\right]-2(0)(\sin 0+0\cos 0)}{\frac{\pi}{2}-0}$$

$$=\frac{\pi\left[0-\frac{\pi}{2}\right]-0}{\frac{\pi}{2}}=-\pi$$

(*Calculus* 7th ed. pages 94–104 / 8th ed. pages 96–106)

23. ANSWER: (E) $f'(x)=3x^2-\frac{1}{x+k} \Rightarrow f'(1)=3-\frac{1}{k+1}=0 \Rightarrow k=-\frac{2}{3}$
(*Calculus* 7th ed. pages 314–323 / 8th ed. pages 322–331)

24. ANSWER: (A)

$$C'(t)=\frac{2(8100+t^2)-2t(2t)}{(8100+t^2)^2}$$
$$=\frac{16{,}200-2t^2}{(8100+t^2)^2}=0 \Rightarrow 16{,}200-2t^2$$
$$=0 \Rightarrow t=\sqrt{8100}=90 \text{ min}$$

To be sure that this value of t occurs when $C(t)$ is a relative maximum, use a sign chart.

t	$0 < t < 90$	$t > 90$
$C'(t)$	positive	negative
$C(t)$	increasing	decreasing

Since $C(t)$ increases before $t = 90$ and decreases after $t = 90$, $C(t)$ has a relative maximum at $t = 90$.
(*Calculus* 7th ed. pages 174–183 / 8th ed. pages 179–189)

25. ANSWER: (C) If $\lim\limits_{x\to\infty}\frac{ae^x}{2+e^x}=3$ and $\lim\limits_{x\to\infty}e^x=\infty$, then

$$\lim_{x\to\infty}\frac{\frac{ae^x}{e^x}}{\frac{2+e^x}{e^x}}=\lim_{x\to\infty}\frac{a}{\frac{2}{e^x}+1}=\frac{a}{1}=3.\ \text{Thus, } a=3.$$

Although l'Hôpital's Rule is not a topic for the AB exam, the solution for this problem can be found using l'Hôpital's Rule (if the student is familiar with it) as follows: $\lim\limits_{x\to\infty}\frac{f(x)}{g(x)}=\lim\limits_{x\to\infty}\frac{f'(x)}{g'(x)}$ can also be used to evaluate a limit when of the form $\frac{\infty}{\infty}$ or $\frac{0}{0}$. Thus,

$\lim_{x\to\infty}\frac{ae^x}{2+e^x}=\lim_{x\to\infty}\frac{ae^x}{e^x}\Rightarrow\lim_{x\to\infty}a=3$, and therefore, $a=3$.

(*Calculus* 7th ed. pages 192–210 / 8th ed. pages 198–208)

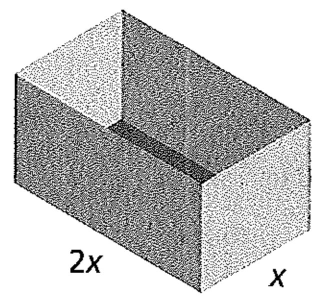
26. ANSWER: (C)

$$\frac{d}{dx}\left[f\left(g(x^2)\right)\right]=f'(g(x^2))\cdot g'(x^2)\cdot 2x\Rightarrow f'(g(4))\cdot g'(4)\cdot 4$$

$$=f'(1)\cdot\left(-\frac{1}{2}\right)\cdot 4=\frac{2}{3}(-2)=-\frac{4}{3}$$

(*Calculus* 7th ed. pages 127–136 / 8th ed. pages 130–140)

27. ANSWER: (B) $V=2x^2h=8\Rightarrow h=\frac{8}{2x^2}=\frac{4}{x^2}$



$$SA=2(2x+x)h+2x^2$$
$$=6x\left(\frac{4}{x^2}\right)+2x^2$$
$$=24x^{-1}+2x^2$$

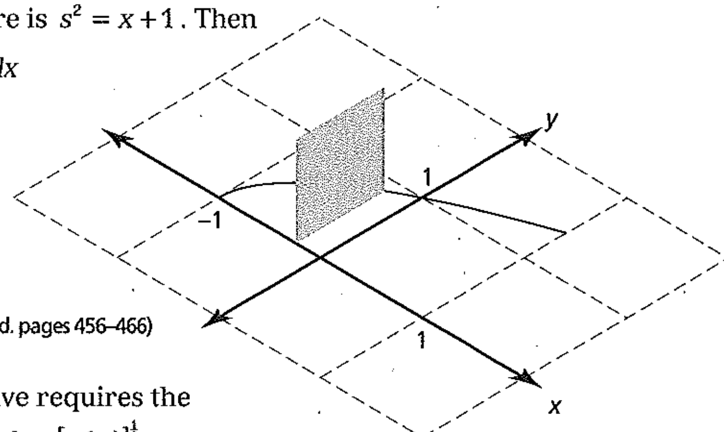
$SA'=-24x^{-2}+4x=0\Rightarrow 4\left(x-\frac{6}{x^2}\right)=0$ and

$x=\sqrt[3]{6}$ is the width of the base. $SA''=48x^{-3}+4>0$ for all x, and the function SA will be concave up, thus $x=\sqrt[3]{6}$ occurs at a relative minimum.

(*Calculus* 7th ed. pages 211–221 / 8th ed. pages 218–228)

28. ANSWER: (D) The length of the side of each square is $s=\sqrt{x+1}$ and the area of each square is $s^2=x+1$. Then

$$V=\int_{-1}^{1}s^2\,dx\Rightarrow\int_{-1}^{1}(x+1)\,dx$$
$$=\left(\frac{x^2}{2}+x\right)\Bigg|_{-1}^{1}$$
$$=\left(\frac{1}{2}+1\right)-\left(\frac{1}{2}-1\right)$$
$$=2$$



(*Calculus* 7th ed. pages 421–431 / 8th ed. pages 456–466)

29. ANSWER: (A) Each derivative requires the use of the Chain Rule. For $h=[g(x)]^{\frac{1}{2}}$,

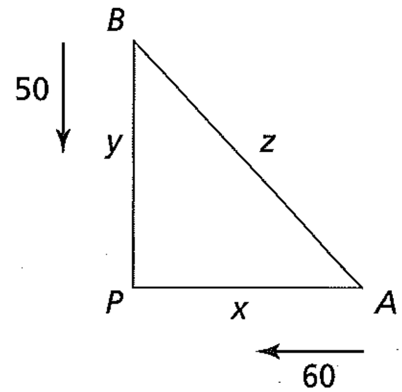
$h'=\frac{1}{2}[g(x)]^{-\frac{1}{2}}\cdot g'(x)$. And

$$h''=\left\{-\frac{1}{4}[g(x)]^{-\frac{3}{2}}\cdot g'(x)\right\}g'(x)+g''(x)\left\{\frac{1}{2}[g(x)]^{-\frac{1}{2}}\right\}.$$

Finally, $h''(0) = \left[-\frac{1}{4}(4)^{-\frac{3}{2}}(8)\right](8) + (-12)\left[\frac{1}{2}(4)^{-\frac{1}{2}}\right] = -2 - 3 = -5$.
(*Calculus* 7th ed. pages 127–136 / 8th ed. pages 130–140)

30. ANSWER: (B) $g'(x) = \int_0^3 g''(x)dx + (-1) \approx 0.527 + (-1) = -0.473$
(*Calculus* 7th ed. pages 275–287 / 8th ed. pages 282–294)

31. ANSWER: (D) Let x = the distance from car A to point P, y = the distance from car B to point P, and z = the distance between the cars at time t.



Then, $\frac{d}{dt}\left(x^2 + y^2 = z^2\right) = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$, which simplifies to $x\frac{dx}{dt} + y\frac{dy}{dt} = z\frac{dz}{dt}$. By substitution, $12(-60) + 10(-50) = \sqrt{12^2 + 10^2}\frac{dz}{dt} \Rightarrow \frac{dz}{dt} \approx -78.102$ Therefore, the distance between the cars is decreasing at approximately 78.102 miles per hour.
(*Calculus* 7th ed. pages 144–152 / 8th ed. pages 149–157)

32. ANSWER: (D) $f'(x) = 2^{-2x}(-2\ln 2) = -1 \Rightarrow x \approx 0.2356$
$f(0.2356) \approx 0.7213$ and the tangent is $y - 0.7213 = -1(x - 0.2356)$ with the x-intercept of $x \approx 0.957$.
(*Calculus* 7th ed. pages 144–152 / 8th ed. pages 149–157)

33. ANSWER: (D) Tangents are parallel when derivatives are equal; $g'(x) = y' \Rightarrow \sec^2(x+2) = 2$. There are five points in the open interval $-4 < x < 5$ where the derivatives intersect. Therefore the tangents to $g(x)$ are parallel to the given line five times in this interval.
(*Calculus* 7th ed. pages 275–287 / 8th ed. pages 282–294)

34. ANSWER: (B)

$$\int_1^{2k}\left(x - \frac{k}{x^2}\right)dx = 15 \Rightarrow \left(\frac{x^2}{2} + \frac{k}{x}\right)\Bigg|_1^{2k}$$
$$= \left(\frac{4k^2}{2} + \frac{k}{2k}\right) - \left(\frac{1}{2} + \frac{k}{1}\right) \Rightarrow 2k^2 + \frac{1}{2} - \frac{1}{2} - k$$
$$= 15 \Rightarrow 2k^2 - k - 15 = 0$$

Factor the quadratic and solve for k. $(2k+5)(k-3) = 0$, and $k = 3$ and $-\frac{5}{2}$. The sum of both k values is $\frac{1}{2}$.
(*Calculus* 7th ed. pages 275–287 / 8th ed. pages 282–294)

35. ANSWER: (D) Use a sign chart to analyze the curve.

	$-\infty < x < -2$	$-2 < x < -1.5$	$-1.5 < x < 0$	$x > 0$
$f'(x)$	positive	negative	negative	positive
$f''(x)$	negative	negative	positive	positive
$f(x)$	increasing concave down	decreasing concave down	decreasing concave up	increasing concave up

Therefore, a correct graph of $f(x)$ is D.
(*Calculus* 7th ed. pages 202–210 / 8th ed. pages 209–217)

36. ANSWER: (C) By the Trapezoidal Rule,

$$\int_0^4 H(t)dt \approx \frac{b-a}{2n}[H(0)+2H(1)+2H(2)+2H(3)+H(4)]$$

where $\frac{b-a}{2n}=\frac{4-0}{2(4)}=\frac{1}{2}$. If $H_{avg}=\frac{1}{b-a}\int_a^b H(t)\ dt$ then H_{avg} is approximated by

$$H_{avg}=\frac{1}{4}\int_0^4 H(t)\ dt \Rightarrow \frac{1}{4}\left\{\frac{1}{2}[0+2(1.3)+2(1.5)+2(2.1)+2.6]\right\}.$$

(*Calculus* 7th ed. pages 300–306 / 8th ed. pages 309–315)

37. ANSWER: (E) Distance traveled, $x(t)=\int_0^{1.5}|v(t)|\ dt \approx 31.669$ cm.
(*Calculus* 7th ed. pages 275–287 / 8th ed. pages 282–294)

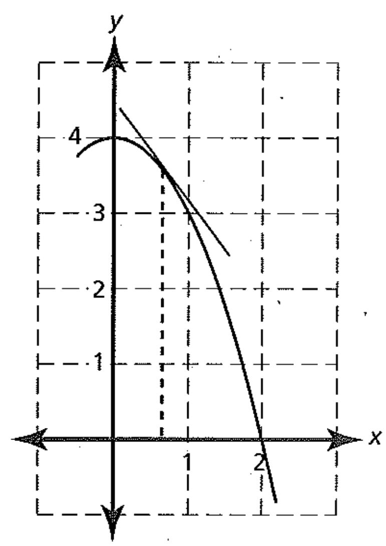
38. ANSWER: (B) I. (False) The Mean Value Theorem is satisfied in the interval $0 < x < 2$, but not necessarily at $c = 1$. (The statement is true if $f(x)=4-x^2$ but not true for a function such as

$f(x)=4-\frac{1}{4}x^4$, where

$f'(c)=-c^3=\frac{f(2)-f(0)}{2-0} \Rightarrow -c^3=\frac{0-4}{2} \Rightarrow c=\sqrt[3]{2}$.)

II. (True) Since $f(x)$ is decreasing and concave down, the area from $x = 0$ to $x = 1$ is always greater than from $x = 1$ to $x = 2$. Use right or left Riemann sums to visualize the statement.

III. (False) The tangent will always be above the curve in this interval and will therefore overestimate the function value.



(*Calculus* 7th ed. pages 168–173, 252–264, 228–234 / 8th ed. pages 172–178, 259–270, 233–247)

39. ANSWER: (E) $f(-2)=(-2)^2+\int_{-2}^{-2} g(t)\ dt = 4+0=4$
(*Calculus* 7th ed. pages 265–274 / 8th ed. pages 271–281)

40. ANSWER: (A) $f'(x)=2x+g(x) \Rightarrow f'(-2)=2(-2)+g(-2)=-4-2=-6$
(*Calculus* 7th ed. pages 275–287 / 8th ed. pages 282–294)

41. ANSWER: (D) $f''(x) = 2 + g'(x) \Rightarrow f''(2) = 2 + g'(2) = 2 + 0 = 2$.
(*Calculus* 7th ed. pages 275–287 / 8th ed. pages 282–294)

42. ANSWER: (E) The curves intersect at $x \approx 1.781$ and the area between the curves is found by $\int_0^{1.781} (f-g)\, dx + \int_{1.781}^{\frac{\pi}{2}} (f-g)\, dx$ $\approx 1.1661 + 0.5953 \approx 1.761$. (The area can also be computed by evaluating the absolute value of the difference of the functions as the functions intersect in the interval $0 \le x \le \frac{\pi}{2}$:

$\int_0^{\frac{\pi}{2}} |f(x) - g(x)|\, dx \approx 1.761$.)
(*Calculus* 7th ed. pages 412–420 / 8th ed. pages 446–455)

43. ANSWER: (C) $f(x)$ is decreasing and concave down when $f'(x)$ and $f''(x)$ are both less than 0.

x	$-8 < x < -6$	$-6 < x < 4.5$	$-4.5 < x < -2$	$-2 < x < 0$	$0 < x < 3$	$3 < x < 5$
f'	negative	positive	positive	negative	negative	negative
f''	positive	positive	negative	negative	positive	negative
f	decreasing	increasing	increasing	decreasing	decreasing	decreasing
	concave up	concave up	concave down	concave down	concave up	concave down

Therefore, the intervals are $-2 < x < 0$ and $3 < x < 5$.
(*Calculus* 7th ed. pages 184–191 / 8th ed. pages 190–197)

44. ANSWER: (A) Since the rate of decay is proportional to the amount present, then $y = Ce^{kt}$. When $t = 0$, $C = 25$. With a half-life of 276 days, solve for k. $12.5 = 25e^{k(276)} \Rightarrow k \approx -0.0025$ and $y = 25e^{-0.0025t}$. Then $y'(t) = 25e^{-0.0025t}(-0.0025)$ and $y'(120) = 25e^{-0.0025(120)}(-0.0025)$ ≈ -0.046 grams per day.
(*Calculus* 7th ed. pages 361–368 / 8th ed. pages 413–420)

45. ANSWER: (D) When rotating about the line $x = 2$, the radius of rotation $= 2 - x$. Rewrite the function for x in terms of y. Then

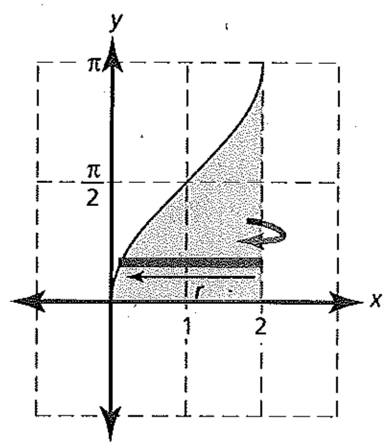
$y = \sin^{-1}(x-1) + \frac{\pi}{2} \Rightarrow y - \frac{\pi}{2}$

$= \sin^{-1}(x-1) \Rightarrow \sin\left(y - \frac{\pi}{2}\right) + 1$

$= x$.

Finally, $r = 2 - x \Rightarrow 2 - \left[\sin\left(y - \frac{\pi}{2}\right) + 1\right]$

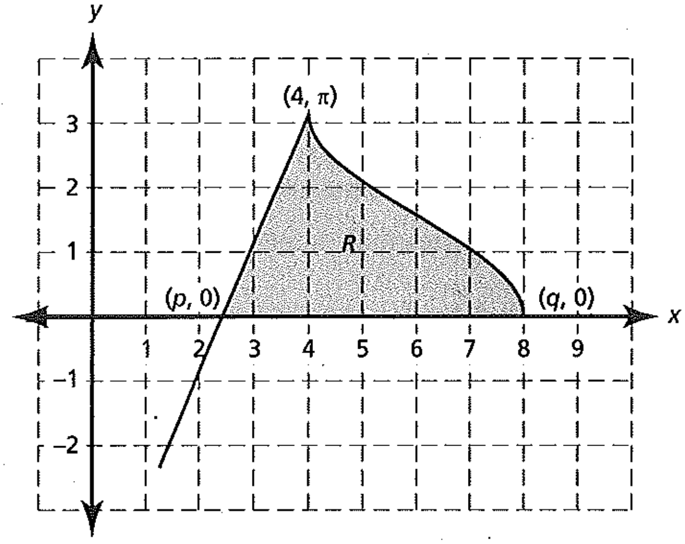
$= 1 - \sin\left(y - \frac{\pi}{2}\right)$.



(*Calculus* 7th ed. pages 421–431 / 8th ed. pages 456–466)

FREE-RESPONSE QUESTIONS

1. Region R is bounded by the functions $f(x)=2(x-4)+\pi$, $g(x)=\cos^{-1}\left(\frac{\pi}{2}-3\right)$, and the x-axis as shown in the figure to the right.
   a. What is the area of region R?
   b. Find the volume of the solid generated when region R is rotated about the x-axis.
   c. Find all values c for $f(x)$ and $g(x)$ in the closed interval $p \le c \le q$ for which each function equals the average value in the indicated interval.



	Solution	Possible points
a.	Root of $f(x)$ is $p = 2.4292$ and the root of $g(x)$ is $q = 8$. Area of $R = \int_{2.4292}^{4} [2(x-4)+\pi]\, dx + \int_{4}^{8}\left[\cos^{-1}\left(\frac{x}{2}-3\right)\right] dx \approx 8.751$	3: $\begin{cases} \text{1: limits and integrand for left branch, } f(x) \\ \text{1: limits and integrand for right branch, } g(x) \\ \text{1: answer} \end{cases}$
b.	Volume = $\pi\left\{\int_{2.4292}^{4}[2(x-4)+\pi]^2\, dx + \int_{4}^{8}\left[\cos^{-1}\left(\frac{x}{2}-3\right)\right]^2 dx\right\}$ $V \approx 16.907\pi$ or 53.115	3: $\begin{cases} \text{1: limits on both integrals} \\ \text{1: both integrands} \\ \quad <-1> \text{ for any error} \\ \text{1: answer} \end{cases}$
c.	The average value of a function on an interval is given by $h(c)=\frac{1}{b-a}\int_a^b h(x)\, dx$, where $a \le c \le b$. Use the answer from part a above. For the left branch, $f(x)$: $2(c_1-4)+\pi=\frac{1}{8-2.4292}[8.751]$ $\Rightarrow c_1 \approx 3.215\ (p < c_1 < 4)$ For the right branch of $g(x)$: $\cos^{-1}\left(\frac{c_2}{2}-3\right)=\frac{1}{8-2.4292}(8.751)$ $\Rightarrow c_2 = 6\ (4 < c_2 < 8)$	3: $\begin{cases} \text{1: use of } f_{avg} \\ \text{1: } c_1 \text{ for left branch, } f(x) \\ \text{1: } c_2 \text{ for right branch, } g(x) \end{cases}$

1. a (*Calculus* 7th ed. pages 275–287 / 8th ed. pages 282–294)

1. b (*Calculus* 7th ed. pages 421–431 / 8th ed. pages 456–466)

1. c (*Calculus* 7th ed. pages 275–287 / 8th ed. pages 282–294)

2. A particle is moving along the x-axis with velocity $v(t) = \ln(x+3) - e^{\frac{x}{2}-1}(\cos x)$, for $0 \le t \le 8$. The initial position of the particle is -1.6.
   a. At what time(s) in the open interval $0 < t < 8$ does the particle change direction? Justify your answer.
   b. Where is the particle when it is farthest to the left?
   c. How far does the particle travel in the interval $3 \le t \le 6$?
   d. At what times in the closed interval $0 \le t \le 8$ is the speed of the particle decreasing? Justify your answer.

<table>
<tr><th></th><th>Solution</th><th>Possible points</th></tr>
<tr><td>a.</td><td>The particle changes direction when $v(t)$ changes from positive to negative or negative to positive. Graph $v(t)$, then $v(t) = 0$ at $t \approx 5.160$ and $t \approx 7.718$.
<table>
<tr><td>t</td><td>$0 < t < 5.160$</td><td>$5.160 < t < 7.718$</td><td>$7.718 < t < 9$</td></tr>
<tr><td>$v(t)$</td><td>positive</td><td>negative</td><td>positive</td></tr>
</table>
The particle moves to the right from 0 to 5.160 sec, to the left from 5.160 to 7.718 sec, and to the right from 7.718 to 8 sec.</td><td>$2: \begin{cases} 1: \text{ values} \\ 1: \text{ reason} \end{cases}$</td></tr>
<tr><td>b.</td><td>$x(t) = x(0) + \int_0^{7.718} v(t)\, dt$
$x(7.718) \approx -1.6 - .2167 \approx -1.817$
The particle is 1.817 units to the left of the origin at $t = 7.718$.</td><td>$2: \begin{cases} 1: \text{ integrand with limits} \\ 1: \text{ answer, including } x(0) \end{cases}$</td></tr>
<tr><td>c.</td><td>Distance $= \int_3^6 |v(t)|\, dt \approx 8.455$</td><td>$2: \begin{cases} 1: \text{ integrand, with limits} \\ 1: \text{ answer} \end{cases}$</td></tr>
<tr><td>d.</td><td>The speed of the particle is decreasing when the velocity and acceleration have opposite signs.
<table>
<tr><th>t</th><th>$v(t)$</th><th>$a(t)$</th></tr>
<tr><td>$0 < t < 3.664$</td><td>positive</td><td>positive</td></tr>
<tr><td>$3.664 < t < 5.160$</td><td>positive</td><td>negative</td></tr>
<tr><td>$5.160 < t < 6.738$</td><td>negative</td><td>negative</td></tr>
<tr><td>$6.738 < t < 7.718$</td><td>negative</td><td>positive</td></tr>
<tr><td>$7.718 < t < 8$</td><td>positive</td><td>positive</td></tr>
</table>
Therefore, the speed of the particle is decreasing in the intervals $3.664 < t < 5.160$ and $6.738 < t < 7.718$.</td><td>$3: \begin{cases} 1: v(t) \text{ and } a(t) \text{ have opposite signs} \\ 1: \text{ analysis} \\ 1: \text{ answers} \end{cases}$</td></tr>
</table>

2. a (*Calculus* 7th ed. pages 174–183 / 8th ed. pages 179–189)

2. b (*Calculus* 7th ed. pages 275–287 / 8th ed. pages 282–294)

2. c (*Calculus* 7th ed. pages 275–287 / 8th ed. pages 282–294)

2. d (*Calculus* 7th ed. pages 117–126 / 8th ed. pages 119–129)

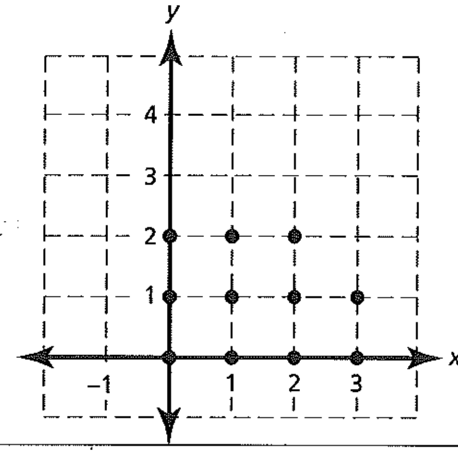
3. A large flea market is held at the local fairgrounds on the first Saturday of each month. The rates at which people enter and leave the fairgrounds are recorded for a 3-hour period beginning when the market is open to the public. The rate at which people arrive is modeled by the function $A(t) = 45\sin(0.03t - .7) + 71$. The function $L(t) = 42\sin(0.034t - 1.52) + 42$ models the rate at which people leave the fairgrounds. Both $A(t)$ and $L(t)$ are measured in people per minute and t is measured for $0 \le t \le 180$ minutes. When the count begins at $t = 0$, there are already 1572 people in the flea market area of the fairgrounds.
   a. How many additional people arrive for the flea market during the 3-hour period after it opens to the public?
   b. Write an expression for $P(t)$, the total number of people at the flea market at time t.
   c. Find the value of $P'(75)$ and explain its meaning.
   d. For $0 \le t \le 180$, at what time is the rate of change of people at the flea market at a maximum? What is the maximum rate of change? Justify your answers.

	Solution	Possible points
a.	$\int_0^{180} A(t)dt = 13{,}945.84$ $\approx 13{,}945$ or $13{,}946$ people	2: $\begin{cases} 1: \text{integral with limits} \\ 1: \text{answer} \end{cases}$
b.	$P(t) = 1572 + \int_0^t [A(x) - L(x)]\, dx$	2: $\begin{cases} 1: \text{integral, with limit in terms of } t \\ 1: \text{answer includes } P(0) \text{ or } 1572 \end{cases}$
c.	$P'(75)$ is the rate at which the number of people arriving and leaving the fairgrounds is changing 75 minutes after the flea market is open to the public. $P'(t) = A(t) - L(t)$ $P'(75) = A(75) - L(75)$ $P'(75) \approx 115.990 - 78.007 = 37.984$ The rate is approximately 37 or 38 people per minute and the number of people at the flea market is increasing because $P'(75) > 0$.	2: $\begin{cases} 1: \text{explanation} \\ 1: \text{value of } P'(75) \end{cases}$
d.	To maximize $P'(t)$, $P''(t) = A'(t) - L'(t) = 0 \Rightarrow t \approx 32.255$ or 127.319. Since the interval is closed, check the value of $P'(t)$ at these two values as well as at the end points of the closed interval. t: 0, 32.255, 127.319, 180 $P'(t)$: 41.956, 58.155 (maximum), 16.272 (minimum), 25.738 The maximum rate is 58 or 59 people per minute at $t = 32.255$ minutes.	3: $\begin{cases} 1: t = 32.255 \\ 1: P'(32.255) \\ 1: \text{reason} \end{cases}$

3. a, b, c (*Calculus* 7th ed. pages 275–287 / 8th ed. pages 282–294)

3. d (*Calculus* 7th ed. pages 174–183 / 8th ed. pages 179–189)

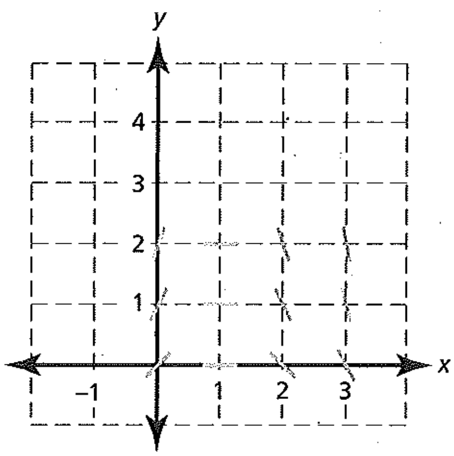
4. Consider the differential equation
$$\frac{dy}{dx} = (y+1)(1-x) \text{ for } y > -1.$$
   a. On the axes provided, sketch a slope field for the given differential equation at the 11 points indicated.
   b. If $y(0) = 1$, then find the particular solution $y(x)$ to the given differential equation.
   c. Draw a function through the point (0, 1) on your slope field which represents an approximate solution of the given differential equation with initial condition $y(0) = 1$.



Solution | **Possible points**

a. The values of dy/dx at the points indicated are given in the table and the sketch below.

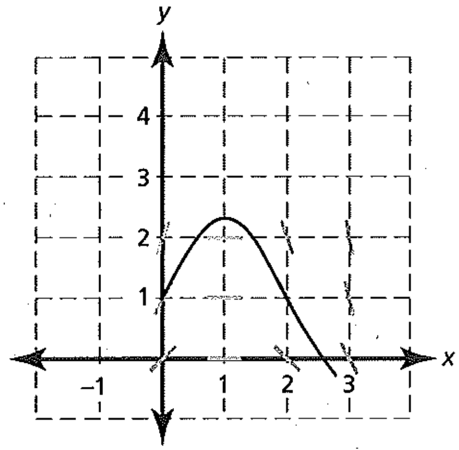
$y \backslash x$	0	1	2	3
2	3	0	–3	–6
1	2	0	–2	–4
0	1	0	–1	–2



Possible points — 2:
- 1: zero slope at each point (x,y) where $x = 1$
- 1: positive slopes at each point (x,y) where $x = 0$; negative slopes at each point (x,y) where $x = 2$ or 3

b.
$$\frac{dy}{y+1} = (1-x)\,dx \Rightarrow$$
$\ln|y+1| = x - \dfrac{x^2}{2} + c_1$. Recall that $y > -1$, so
$$y+1 = e^{x-\frac{x^2}{2}+c_1} = e^{x-\frac{x^2}{2}}e^{c_1} = Ce^{x-\frac{x^2}{2}}.$$
Then, $y = -1 + Ce^{x-\frac{x^2}{2}}$. Using the initial condition (0, 1), $C = 2$ and $y = -1 + 2e^{x-\frac{x^2}{2}}$.

Possible points — 5:
- 1: separating variables
- 1: antiderivatives
- 1: constant of integration
- 1: uses initial condition
- 1: solves for y; 0/1 if y is not exponential

	Solution	Possible points
c.	 $y=-1+2e^{x-\frac{x^2}{2}}$	$2:\begin{cases}1: \text{maximum at } x=1 \text{ only} \\ \quad\ \text{curve above } x\text{-axis} \\ 1: \text{following slopes from} \\ \quad\ \text{part a}\end{cases}$

4. a, b, c (*Calculus* 7th ed. pages A2–A3 / 8th ed. pages 404–408)

5. Consider the function $h(x)=3x^2-\sqrt{x+1}$.
   a. Evaluate $\frac{1}{3-(-1)}\int_{-1}^{3}\left(3x^2-\sqrt{x+1}\right)dx$ and interpret its meaning.
   b. What is the equation of the tangent to $h(x)$ at $x = 0$?
   c. Use the tangent found in part b to approximate $h(x)$ at $x=-0.01$.
   d. Is the approximation, found in part c, greater or less than the actual value of $h(x)$ at $x=-0.01$? Justify your answer using calculus.

	Solution	Possible points
a.	Rewrite the given integral: $\frac{1}{4}\int_{-1}^{3}\left(3x^2-(x+1)^{\frac{1}{2}}\right)dx$ $=\frac{1}{4}\left\{x^3-\left(\frac{2}{3}\right)\left[(x+1)^{\frac{3}{2}}\right]\right\}\Big\|_{-1}^{3}$ $=\frac{1}{4}\left[27-\frac{2}{3}(8)-(-1-0)\right]$ $=\frac{17}{3}$ The integral represents the average value of the function in the interval $-1\le x\le 3$.	$4:\begin{cases}2: \text{antiderivatives} <-1> \text{ for each error} \\ 1: \text{value of integral expression} \\ 1: \text{recognizing average value of } h(x) \text{ on interval } -1\le x\le 3\end{cases}$

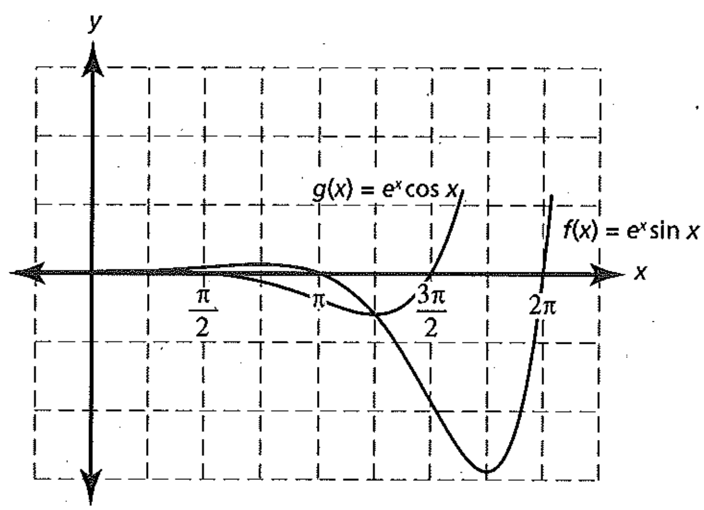
	Solution	Possible points
b.	$h(x)=3x^2-(x+1)^{\frac{1}{2}}$ and $h(0)=-1$ $h'(x)=6x-\left[\left(\frac{1}{2}\right)(x+1)^{-\frac{1}{2}}\right]$ $h'(0)=0-\frac{1}{2}(1)^{-\frac{1}{2}}=-\frac{1}{2}$ Since $h(0)=1$ and $h'(0)=-\frac{1}{2}$, then the tangent line to $h(x)$ at $x=0$ is $y+1=-\frac{1}{2}(x)\Rightarrow y=-\frac{1}{2}(x)-1$.	$2: \begin{cases} 1: & h'(0) \\ 1: & \text{equation of tangent line} \end{cases}$
c.	$h(-.01)\approx-\frac{1}{2}(-0.01)-1=-0.995$	1: answer
d.	$h''(x)=6+\frac{1}{4}(x+1)^{-\frac{3}{2}}$ and $h''(x)>0$ for all x. Therefore $h(x)$ is concave up at $x=0$ and the tangent line will be below the curve. Thus the value of the approximation near $x=0$ will be less than the actual value.	$2: \begin{cases} 1: & \text{finding } h''(x) \\ 1: & \text{reason for under approximation} \end{cases}$

5. a (*Calculus* 7th ed. pages 275–287 / 8th ed. pages 282–294)

5. b (*Calculus* 7th ed. pages 127–136 / 8th ed. pages 130–140)

5. c, d (*Calculus* 7th ed. pages 228–234 / 8th ed. pages 235–247)

6. Consider the functions $f(x)=e^x\sin x$ and $g(x)=e^x\cos x$, as shown in the sketch to the right, in the closed interval $0\le x\le 2\pi$.



a. Let $D(x)=f(x)-g(x)$ be the vertical distance between the functions. Find the value of x in the open interval $\frac{\pi}{4}<x<\frac{5\pi}{4}$, where $D(x)$ is a maximum value. Explain your reasoning.

b. At what value of x in the open interval $\frac{\pi}{4}<x<\frac{5\pi}{4}$ is the rate of change of $D(x)$ increasing the most rapidly? Explain your reasoning.

c. For $H(x)=\frac{e^x}{\sin x}+g(x)$, find the x-coordinate of all points at which $H(x)$ has horizontal tangents on the open interval $0<x<2\pi$.

<table>
<tr><th></th><th>Solution</th><th>Possible points</th></tr>
<tr><td>a.</td><td>

$D(x)=f(x)-g(x)$
$D'(x)=f'(x)-g'(x)$
$D'(x)=e^x\sin x+e^x\cos x-(e^x\cos x-e^x\sin x)$
$=2e^x\sin x=0\Rightarrow x=\pi$

<table>
<tr><td>x</td><td>$\frac{\pi}{4}<x<\pi$</td><td>$\pi<x<\frac{5\pi}{4}$</td></tr>
<tr><td>$D'(x)$</td><td>positive</td><td>negative</td></tr>
<tr><td>$D(x)$</td><td>increasing</td><td>decreasing</td></tr>
</table>

Therefore, $D(x)$ is a maximum at $x=\pi$, because $D(x)$ is increasing to the left of $x=\pi$ and decreasing to the right of $x=\pi$, which is the only critical value in the interval $\frac{\pi}{4}<x<\frac{5\pi}{4}$.

</td><td>

$3:\begin{cases}1: & \text{find } D'(x)\\ 1: & \text{set } D'(x)=0\\ 1: & \text{answer and reason}\end{cases}$

</td></tr>
<tr><td>b.</td><td>

The rate of change of $D(x)$ is given by $D'(x)$, and the absolute maximum of $D'(x)$ occurs when $D''(x)=0$.
$D''(x)=2e^x\sin x+2e^x\cos x=0$
$=2e^x(\sin x+\cos x)=0$
$\Rightarrow\sin x=-\cos x\Rightarrow x=\frac{3\pi}{4}$.

<table>
<tr><td>x</td><td>$\frac{\pi}{4}<x<\frac{3\pi}{4}$</td><td>$\frac{3\pi}{4}<x<\frac{5\pi}{4}$</td></tr>
<tr><td>$D''(x)$</td><td>positive</td><td>negative</td></tr>
<tr><td>$D'(x)$</td><td>increasing</td><td>decreasing</td></tr>
</table>

Therefore, $D'(x)$, the rate of change in the distance, is increasing most rapidly (an absolute maximum) at $x=\frac{3\pi}{4}$ because $D'(x)$ is increasing to the left of $x=\frac{3\pi}{4}$ and decreasing to the right of $x=\frac{3\pi}{4}$, which is the only critical value in the interval $\frac{\pi}{4}<x<\frac{5\pi}{4}$.

</td><td>

$3:\begin{cases}1: & \text{find } D''(x)\\ 1: & \text{set } D''(x)=0\\ 1: & \text{answer with reason}\end{cases}$

</td></tr>
</table>

	Solution	Possible points
c.	$H(x)=\dfrac{e^x}{\sin x}+g(x)$ $H'(x)=\dfrac{e^x\sin x-e^x\cos x}{\sin^2 x}+e^x\cos x-e^x\sin x$ $\Rightarrow \dfrac{e^x(\sin x-\cos x)-e^x\sin^2 x(\sin x-\cos x)}{\sin^2 x}$ $\Rightarrow \dfrac{e^x(\sin x-\cos x)(1-\sin^2 x)}{\sin^2 x}=0$ In the open interval $0<x<2\pi$, the horizontal tangents occur when $H'(x)=0$. Then, $1-\sin^2 x=0\Rightarrow x=\dfrac{\pi}{2}$ and $\dfrac{3\pi}{2}$, or $\sin x-\cos x=0\Rightarrow x=\dfrac{\pi}{4}$ and $\dfrac{5\pi}{4}$.	3: $\begin{cases}1: & \text{find derivative of } H(x)\\ 1: & \text{set } H'(x)=0\\ 1: & \text{answers}\end{cases}$

6. a (*Calculus* 7th ed. pages 174–183 / 8th ed. pages 179–189)

6. b (*Calculus* 7th ed. pages 184–191 / 8th ed. pages 190–197)

6. c (*Calculus* 7th ed. pages 117–126 / 8th ed. pages 119–129)

CALCULUS AB AND BC SCORING CHART

SECTION I: MULTIPLE CHOICE

__________ – (__________ × 1/4) × 1.2 = __________ = __________

\# correct (out of 45) | # incorrect | total (out of 54) | (round to nearest whole number)

SECTION II: FREE RESPONSE

Question 1	Score out of 9 points =	__________
Question 2	Score out of 9 points =	__________
Question 3	Score out of 9 points =	__________
Question 4	Score out of 9 points =	__________
Question 5	Score out of 9 points =	__________
Question 6	Score out of 9 points =	__________
	Sum for Section II =	__________ (out of 45)

Composite Score

Section I total	=
Section II total	=
Composite score	= __________ (out of 108)

Grade Conversion Chart*

Composite score range	AP Exam Grade
70–108	5
55–69	4
40–54	3
30–39	2
0–29	1

***Note:** The ranges listed above are only approximate. Each year the ranges for the actual AP Exam are somewhat different. The cutoffs are established after the exams are given to over 200,000 students, and are based on the difficulty level of the exam each year.