CHAB Name_____

Midterm Review

Evaluate each limit.

1) $\lim_{x \to 3} \left(-x^3 + 2x^2 + 2\right)$

A) -9 B) -7
C) 3 D) -1

2) $\lim_{x \to 2^+} \dfrac{2\left|-x + 2\right|}{-x + 2}$

A) 1 B) 7
C) -6 D) -2

3) $\lim_{x \to 1} -\dfrac{x^2 - 4x + 3}{x - 1}$

A) 9 B) -4
C) -5 D) 2

4) $\lim_{x \to 3^-} \dfrac{3}{x - 3}$

A) 10 B) -10
C) ∞ D) $-\infty$

5) $\lim_{x \to -\infty} \dfrac{x}{x - 1}$

A) 0 B) $-\infty$
C) ∞ D) 1

For each problem, find the average rate of change of the function over the given interval.

6) $y = -x^2 + x - 1; \ [0, 2]$

A) $-\dfrac{1}{4}$ B) 0

C) $-\dfrac{1}{2}$ D) -1

Use the definition of the derivative to find the derivative of each function with respect to *x*.

7) $y = -x^2 + 4$

A) $\dfrac{dy}{dx} = -x - 10$

B) $\dfrac{dy}{dx} = -2x$

C) $\dfrac{dy}{dx} = -2x^2 + 9$

D) $\dfrac{dy}{dx} = -2x - 10$

For each problem, find the equation of the tangent line to the function at the given point.

8) $y = -x^2 - 1; \ (-1, -2)$

A) $y = 2x$ B) $y = -x - 3$
C) $y = -2$ D) $y = -6x - 8$

Differentiate each function with respect to *x*.

9) $y = 2x + \frac{4}{5}x^{-1}$

A) $\frac{dy}{dx} = 2 - \frac{4}{5x^2}$

B) $\frac{dy}{dx} = 2 + \frac{4}{5x^2}$

C) $\frac{dy}{dx} = -\frac{8}{5x^2}$

D) $\frac{dy}{dx} = 2x - \frac{4}{5x}$

For each problem, find the indicated derivative with respect to *x*.

10) $y = -2x^5 + x^3$ Find $\frac{d^3y}{dx^3}$

A) $\frac{d^3y}{dx^3} = -120x^2 + 6$

B) $\frac{d^3y}{dx^3} = -250x^5 + 27x^3$

C) $\frac{d^3y}{dx^3} = -2x^2 + 1$

D) $\frac{d^3y}{dx^3} = -7x$

Differentiate each function with respect to *x*.

11) $y = \left(-x^2 + 1\right) \cdot -2x^5$

A) $\frac{dy}{dx} = \left(-x^2 + 1\right) \cdot -2x^5 - 2x \cdot -10x^4$
$= 2x^7 + 18x^5$

B) $\frac{dy}{dx} = \left(-x^2 + 1\right) \cdot -10x^4 - 2x^5 \cdot -2x$
$= 14x^6 - 10x^4$

C) $\frac{dy}{dx} = -2x^5 - 10x^4$

D) $\frac{dy}{dx} = \left(-x^2 + 1\right) \cdot -2x - 2x^5 \cdot -10x^4$
$= 20x^9 + 2x^3 - 2x$

12) $y = \frac{1}{4x^5 - 5}$

A) $\frac{dy}{dx} = -\frac{20x^4}{4x^5 - 5}$
$= -\frac{20x^4}{4x^5 - 5}$

B) $\frac{dy}{dx} = 20x^4$

C) $\frac{dy}{dx} = -20x^4$
$= -20x^4$

D) $\frac{dy}{dx} = -\frac{20x^4}{\left(4x^5 - 5\right)^2}$
$= -\frac{20x^4}{16x^{10} - 40x^5 + 25}$

13) $y = (-3x^4 - 4)^2$

A) $\frac{dy}{dx} = 2(-3x^4 - 4) \cdot -12x^3$
$= -24x^3(-3x^4 - 4)$

B) $\frac{dy}{dx} = -12x^3$

C) $\frac{dy}{dx} = 2(-3x^4 - 4)$

D) $\frac{dy}{dx} = (-3x^4 - 4) \cdot -12x^3$

For each problem, you are given a table containing some values of differentiable functions $f(x)$, $g(x)$ and their derivatives. Use the table data and the rules of differentiation to solve each problem.

14)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	1	1	2	1
2	2	1	3	1
3	3	1	4	0
4	4	1	3	−1

Part 1) Given $h_1(x) = f(x) + g(x)$, find $h_1'(1)$

Part 2) Given $h_2(x) = f(x) - g(x)$, find $h_2'(2)$

A) $h_1'(1) = 2$
$h_2'(2) = 0$

B) $h_1'(1) = 2$
$h_2'(2) = 2$

C) $h_1'(1) = -1$
$h_2'(2) = 0$

D) $h_1'(1) = 3$
$h_2'(2) = 3$

Differentiate each function with respect to *x*.

15) $y = \sin x^5$

A) $\frac{dy}{dx} = \tan x^5 \cdot 5x^4$
$= 5x^4 \tan x^5$

B) $\frac{dy}{dx} = \sec x^5 \cdot 5x^4$
$= 5x^4 \sec x^5$

C) $\frac{dy}{dx} = -\cos x^5 \cdot 5x^4$
$= -5x^4 \cos x^5$

D) $\frac{dy}{dx} = \cos x^5 \cdot 5x^4$
$= 5x^4 \cos x^5$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

16) $4x^2 = -4y^3 + 2$

A) $\frac{dy}{dx} = \frac{2x^2}{-2y^3 + 1}$

B) $\frac{dy}{dx} = -\frac{3y^2}{2x}$

C) $\frac{dy}{dx} = -\frac{4x}{-2y^3 + 1}$

D) $\frac{dy}{dx} = -\frac{2x}{3y^2}$

For each problem, find the derivative of the function at the given value.

17) $y = -(-x + 2)^{\frac{1}{2}}$ at $x = 0$

A) $\left.\frac{dy}{dx}\right|_{x=0} = \frac{\sqrt{5}}{10}$

B) $\left.\frac{dy}{dx}\right|_{x=0} = \frac{\sqrt{3}}{6}$

C) $\left.\frac{dy}{dx}\right|_{x=0} = \frac{1}{2}$

D) $\left.\frac{dy}{dx}\right|_{x=0} = \frac{\sqrt{2}}{4}$

For each problem, find the values of c that satisfy Rolle's Theorem.

18) $f(x) = -2x^2 - 16x - 30; \quad [-5, -3]$

A) $\{-4\}$ B) $\{-5\}$

C) $\left\{-\frac{7}{2}\right\}$ D) $\left\{-\frac{9}{2}\right\}$

For each problem, find the values of c that satisfy the Mean Value Theorem.

19) $y = 2x^2 + 16x + 30; \quad [-6, -2]$

A) $\left\{-\frac{11}{2}\right\}$ B) $\{-4\}$

C) $\left\{-\frac{7}{2}\right\}$ D) $\{-3\}$

For each problem, find the open intervals where the function is increasing and decreasing.

20) $y = \frac{x^2}{2} - x + \frac{3}{2}$

A) Increasing: $\left(-\infty, \frac{1}{3}\right)$ Decreasing: $\left(\frac{1}{3}, \infty\right)$

B) Increasing: $(1, \infty)$ Decreasing: $(-\infty, 1)$

C) Increasing: $(4, \infty)$ Decreasing: $(-\infty, 4)$

D) Increasing: $(-\infty, 1)$ Decreasing: $(1, \infty)$

For each problem, find the open intervals where the function is concave up and concave down.

21) $y = x^2 + 2$

A) Concave up: $(-\infty, \infty)$ Concave down: No intervals exist.

B) Concave up: $(-\infty, 3)$ Concave down: $(3, \infty)$

C) Concave up: No intervals exist. Concave down: $(-\infty, \infty)$

D) Concave up: $(-\infty, 2)$ Concave down: $(2, \infty)$

For each problem, find all points of relative minima and maxima.

22) $f(x) = -2x^2 + 3$

A) No relative minima.
Relative maximum: $(0, 3)$

B) Relative minimum: $\left(\frac{1}{3}, \frac{25}{9}\right)$
No relative maxima.

C) Relative minimum: $(4, -29)$
No relative maxima.

D) No relative minima.
No relative maxima.

For each problem, find all points of absolute minima and maxima on the given interval.

23) $f(x) = -2x^2 - 12x - 18; \quad [-4, -2]$

A) Absolute minima: $(-4, -2), (-2, -2)$
Absolute maximum: $(-3, 0)$

B) No absolute minima.
No absolute maxima.

C) No absolute minima.
Absolute maximum: $(-3, 0)$

D) Absolute minimum: $(-3, 0)$
No absolute maxima.

Solve each optimization problem.

24) A rancher wants to construct two identical rectangular corrals using 400 ft of fencing. The rancher decides to build them adjacent to each other, so they share fencing on one side. What dimensions should the rancher use to construct each corral so that together, they will enclose the largest possible area?

A) 55 ft (non-adjecent sides) by 60 ft (adjacent sides)

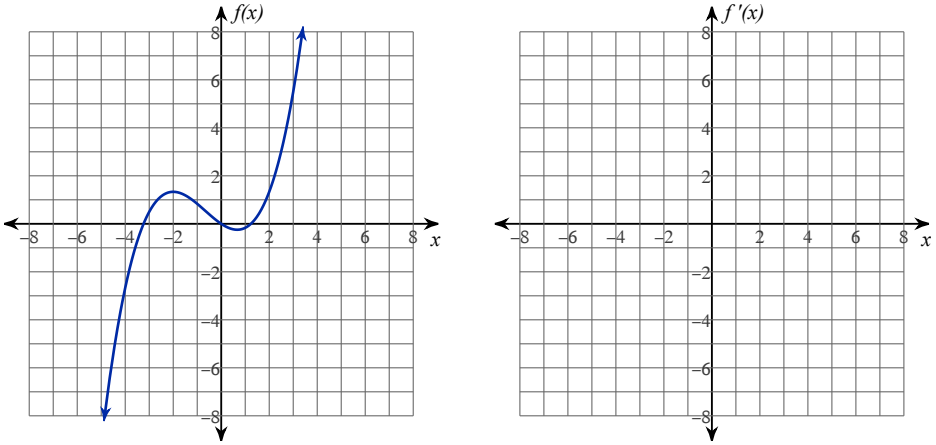
B) 53 ft (non-adjecent sides) by $\frac{188}{3}$ ft (adjacent sides)

C) 51 ft (non-adjecent sides) by $\frac{196}{3}$ ft (adjacent sides)

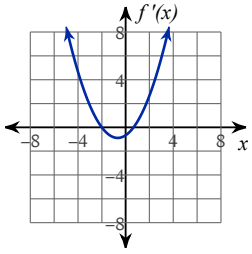
D) 50 ft (non-adjecent sides) by $\frac{200}{3}$ ft (adjacent sides)

Given the graph of $f(x)$, sketch an approximate graph of $f'(x)$.

25)



A)



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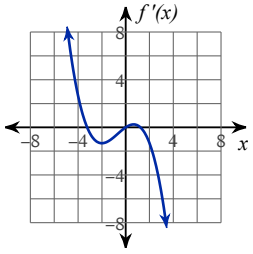

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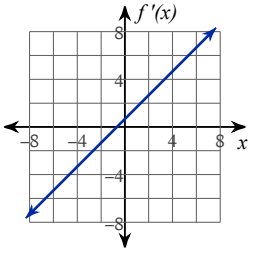

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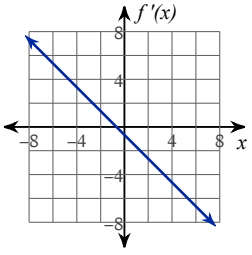


A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the velocity function $v(t)$ and the acceleration function $a(t)$.

26) $s(t) = t^3 - 23t^2 + 120t$

A) $v(t) = 3t^2 - 32t + 64$, $a(t) = 6t - 32$

B) $v(t) = 3t^2 - 2t - 56$, $a(t) = 6t - 2$

C) $v(t) = 3t^2 - 46t + 120$, $a(t) = 6t - 46$

D) $v(t) = 3t^2 - 20t$, $a(t) = 6t - 20$

Solve each related rate problem.

27) An observer stands 700 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of 200 ft/sec. Assume the scenario can be modeled as a right triangle. How fast is the angle of elevation (in radians/sec) from the observer to rocket changing when the rocket is 2400 ft from the ground?

A) $\frac{14}{625}$ radians/sec B) $\frac{14}{629}$ radians/sec

C) $\frac{7}{309}$ radians/sec D) $\frac{1}{78}$ radians/sec

28) A spherical balloon is inflated at a rate of $\frac{32\pi}{3}$ cm³/sec. How fast is the radius of the balloon increasing when the radius is 3 cm?

A) $\frac{1}{18}$ cm/sec B) $\frac{4}{9}$ cm/sec

C) $\frac{8}{27}$ cm/sec D) $\frac{8}{21}$ cm/sec

Evaluate each indefinite integral.

29) $\int -4x^3\,dx$

A) $-4x^4 + C$ B) $-x^4 + C$

C) $-x^3 + C$ D) $-4x + C$

30) $\int \frac{5}{x}\,dx$

A) $\ln|x| + C$ B) $\frac{5 \cdot 2^x}{\ln 2} + C$

C) $2^x + C$ D) $5\ln|x| + C$

31) $\int 3\cos x\,dx$

A) $3\sin x + C$ B) $3\cos x + C$

C) $3\csc x + C$ D) $3\tan x + C$

For each problem, approximate the area under the curve over the given interval using 4 left endpoint rectangles.

32) $y = -x^2 - 2x + 10;\ [-4, 0]$

A) $\frac{91}{3} \approx 30.333$ B) 60

C) $\frac{59}{2} = 29.5$ D) 30

Evaluate each indefinite integral.

33) $\int -\frac{4}{x}\,dx$

A) $-4\ln|x| + C$

B) $e^x + C$

C) $3^x + C$

D) $\ln|x| + C$

34) $\int \frac{1}{\sec x}\,dx$

A) $\csc x + C$ B) $\cos x + C$

C) $\sin x + C$ D) $\tan x + C$

For each problem, find the equation of the line tangent to the function at the given point. Your answer should be in slope-intercept form.

35) $y = (x + 2)^{\frac{1}{2}}$ at $(2, 2)$

A) $y = \frac{1}{4}x + \frac{3}{2}$

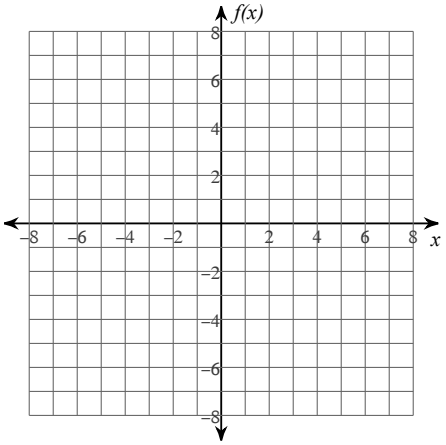
B) $y = \frac{1}{4}x - 3$

C) $y = \frac{1}{5}x + 3$

D) $y = \frac{\sqrt{5}}{10}x + \frac{7\sqrt{5}}{10}$

For each problem, find the: x and y intercepts, x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

36) $f(x) = -2x^3 - 2x^2$



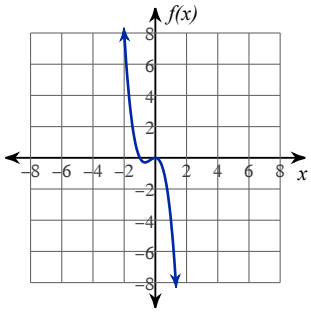
Solve each related rate problem.

37) A conical paper cup is 10 cm tall with a radius of 10 cm. The cup is being filled with water at a rate of $\frac{9\pi}{4}$ cm³/sec. How fast is the water level rising when the water level is 6 cm?

Answers to Midterm Review

1) B
2) D
3) D
4) D
5) D
6) D
7) B
8) A
9) A
10) A
11) B
12) D
13) A
14) A
15) D
16) D
17) D
18) A
19) B
20) B
21) A
22) A
23) A
24) D
25) A
26) C
27) A
28) C
29) B
30) D
31) A
32) D
33) A
34) C
35) A

36)



x-intercepts at $x = -1, 0$ y-intercept at $y = 0$

Critical points at: $x = -\frac{2}{3}, 0$

Increasing: $\left(-\frac{2}{3}, 0\right)$ Decreasing: $\left(-\infty, -\frac{2}{3}\right), (0, \infty)$

Inflection point at: $x = -\frac{1}{3}$

Concave up: $\left(-\infty, -\frac{1}{3}\right)$ Concave down: $\left(-\frac{1}{3}, \infty\right)$

Relative minimum: $\left(-\frac{2}{3}, -\frac{8}{27}\right)$ Relative maximum: $(0, 0)$

37) $\frac{1}{16}$ cm/sec