CHAB - midterm review solns

① $\lim_{x\to 3}(-x^3+2x^2+2) = -(3)^3+2(3)^2+2$

$= -27+2(9)+2$

B $\quad = -27+18+2 = \boxed{-7}$

② $\lim_{x\to 2^+} \frac{2|-x+2|}{-x+2} = \boxed{-2}$ $\quad\boxed{D}$

x	2	2.1	2.2	2.3
y		$\frac{2\|(-.1)\|}{-.1}=-2$	$\frac{2\|-.2\|}{-.2}=-2$	$\frac{2\|-.3\|}{-.3}=-2$

$-2 = y$

③ $\lim_{x\to 1} -\frac{x^2-4x+3}{x-1} = \lim_{x\to 1} -\frac{(x-1)(x-3)}{\cancel{(x-1)}}$

$= \lim_{x\to 1} \frac{-(x-3)}{1} = \frac{-(1-3)}{1} = -(-2) = \boxed{2}$

$\boxed{D}$

④ $\lim_{x\to 3^-} \frac{3}{x-3}$ ← NOTE $x-3=0$, $x=3$ is vertical asymptote ... so ANS will be $\pm\infty$

make a table

0	1	2	3
$\frac{3}{0-3}=-1$	$\frac{3}{1-3}$ $3/-2 = -1.5$	$\frac{3}{2-3} = -3$	VA

$-\infty$

$\lim_{x\to 3^-} \frac{3}{x-3} = \boxed{-\infty}$

D

⑤ $\lim_{x\to -\infty} \frac{x}{x-1}$ ← this is asking about a horizontal asymptote

since numerator and denominator have same degree,

ANS is $y = \frac{\text{lead coef}}{\text{lead coef}} = \frac{1}{1} = \boxed{1}$ D

CHAb-midterm review solns

⑥ $y = -x^2 + x - 1 \quad [0, 2]$

$y(0) = 0 + 0 - 1 = -1 \qquad y(2) = -(2)^2 + 2 - 1$

$= -4 + 2 - 1 = -3$

Avg Rate of change $= \dfrac{y(2) - y(0)}{2 - 0} = \dfrac{-3 - (-1)}{2 - 0} = \dfrac{-2}{2} = \boxed{-1}$

(D)

⑦ $y = -x^2 + 4$

shortcut (since it's multiple choice) just find the derivative w/ the power rule.

$\boxed{y' = -2x}$ (B)

long way (using actual def'n of Derivative)

$y' = \lim\limits_{h \to 0} \dfrac{-(x+h)^2 + 4 - (-x^2 + 4)}{h}$

$= \lim\limits_{h \to 0} \dfrac{-(x^2 + 2xh + h^2) + 4 + x^2 - 4}{h}$

$= \lim\limits_{h \to 0} \dfrac{-x^2 - 2xh - h^2 + 4 + x^2 - 4}{h}$

$= \lim\limits_{h \to 0} \dfrac{-2xh - h^2}{h} = \lim\limits_{h \to 0} \dfrac{(-2x - h)\cancel{h}}{\cancel{h}}$

$= \lim\limits_{h \to 0} -2x - h = -2x - 0 = \boxed{-2x}$

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(8) $y = -x^2 - 1$ $(-1,-2)$ Tangent line EQ

$y' = -2x$

$m_T = -2(-1) = 2 \leftarrow$ so $m_T = 2$ which makes it ANS (A) ...

BUT IF you WANT TO FINISH...

$y - (-2) = 2(x - (-1))$

$y + 2 = 2(x+1)$

$y + 2 = 2x + 2$

$\boxed{y = 2x}$ (A)

---

(9) $y = 2x + \frac{4}{5}x^{-1}$

$y' = 2 - \frac{4}{5}x^{-2} = \boxed{2 - \frac{4}{5x^2}}$ (A)

---

(10) $y = -2x^5 + x^3$ FIND $\frac{d^3y}{dx^3} = y'''$

$y' = -10x^4 + 3x^2$

$y'' = -40x^3 + 6x$

$y''' = -120x^2 + 6$ (A)

---

(11) $y = (-x^2+1) \cdot -2x^5$

OPT 1: product rule

$y = \underset{u}{(-x^2+1)} \cdot \underset{v}{-2x^5}$

$y' = (-x^2+1)(-10x^4) + -2x^5(-2x)$

$y' = 10x^6 - 10x^4 + 4x^6$

$\boxed{y' = 14x^6 - 10x^4}$

OPT 2: simplify FIRST

$y = 2x^7 - 2x^5$

$\boxed{y' = 14x^6 - 10x^4}$

(B)

(12) $y = \frac{1}{4x^5-5}$

OPT 1: QUOTIENT rule

$y = \frac{1}{4x^5-5} \begin{matrix} \leftarrow H \\ \leftarrow L \end{matrix}$

$y' = \frac{(4x^5-5)(0)-(1)(20x^4)}{(4x^5-5)^2}$

$y' = \frac{-20x^4}{(4x^5-5)(4x^5-5)}$

$\boxed{y' = \frac{-20x^4}{16x^{10}-40x^5+25}}$

OPT 2: chAIN rule

$y = (4x^5-5)^{-1}$

$y' = -1(4x^5-5)^{-2}(20x^4)$

$= \frac{-(20x^4)}{(4x^5-5)^2}$

$\boxed{y' = \frac{-20x^4}{16x^{10}-40x^5+25}}$

(D)

(13) $y = (-3x^4-4)^2$

$y' = 2(-3x^4-4)^1(-12x^3) = \boxed{-24x^3(-3x^4-4)}$

(A)

(14) $h_1(x) = f(x)+g(x)$

$h_1'(x) = f'(x)+g'(x)$

$h_1'(1) = f'(1)+g'(1)$

$h_1'(1) = 1+1 = \boxed{2}$

$h_2(x) = f(x)-g(x)$

$h_2'(x) = f'(x)-g'(x)$

$h_2'(2) = f'(2)-g'(2)$

$h_2'(2) = 1-1 = \boxed{0}$

(A)

(15) $y = \sin(x^5)$

$y' = \cos(x^5) \cdot 5x^4$

$\boxed{y' = 5x^4\cos(x^5)}$

(D)

(16) $4x^2 = -4y^3+2$

$8x = -12y^2\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{8x}{-12y^2} = \frac{2x}{-3y^2}$

(D)

CHAB - midterm review solns

(17) $y = -(-x+2)^{1/2}$

$y' = -\frac{1}{2}(-x+2)^{-1/2}(-1) = \frac{1}{2}(-x+2)^{-1/2}$

$y' = \frac{1}{2\sqrt{-x+2}}$ $\quad y'(0) = \frac{1}{2\sqrt{0+2}} = \frac{1}{2\sqrt{2}}$

$y'(0) = \frac{1}{2\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{4}}$ (D)

(18) to use Rolle's thm, the fxn needs to meet three requirements on $[a,b]$

(1) $f(x)$ is continuous (yes, polynomial) on $[a,b]$

(2) $f(x)$ is differentiable (yes, polynomial) on (a,b)

(3) $f(a) = f(b)$ (yes, $f(-5) = -2(25) - 16(-5) - 30$
$= -50 + 80 - 30 = 0$

$f(-3) = -2(9) - 16(-3) - 30$
$-18 + 48 - 30 = 0$)

$0 = 0$

Rolle's is basically a specific version of MVT
avg slope (which $= 0$ since $f(a) = f(b)$) = instant slope

$m = \frac{f(b) - f(a)}{b - a} = \frac{0-0}{-5+3} = 0$ $\quad f'(x) = -4x - 16$

$0 \leftrightarrow -4x - 16$

$16 = -4x$

$\boxed{x = -4}$

(A) $\{-4\}$

CHAB - midterm review solns

(19) to use MVT,

① $f(x)$ continuous on $[-6,-2]$ ← yes, polynomial

② $f(x)$ Differentiable on $(-6,-2)$ ← yes, Differentiable

then...

Avg slope = instant slope

$f(-6) = 2(36) + 16(-6) + 30$

$= 72 + -96 + 30 = 6$

$f(-2) = 2(4) + 16(-2) + 30$

$= 8 - 32 + 30 = 6$

Avg slope

$\frac{f(-6) - f(-2)}{-6+2}$

$= \frac{6-6}{-6+2}$

$= 0$

$f'(x) = 4x + 16$

$= 4x + 16$

$-16 = 4x$

$\boxed{-4 = x}$

B

---

(20) $y = \frac{x^2}{2} - x + \frac{3}{2}$

$y' = x - 1$

CRIT. pts

$y' = 0$

$x - 1 = 0$

$x = 1$

y' DNE never

sign chart

	$-\infty$		1		∞
$x-1$		$-$	0	$+$	
y'		$\ominus$		$\oplus$	
y		DEC ↘		INC ↗	

INC $(1, \infty)$

Dec $(-\infty, 1)$

B

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(21) $y = x^2 + 2$

$y' = 2x$

$y'' = 2$ ← so $y'' > 0$ always...

thus y is concave up $(-\infty, \infty)$ and never concave down (A)

(22) $f(x) = -2x^2 + 3$

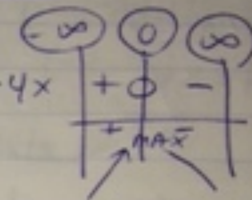
$f'(x) = -4x$

crit pts

$f'(x) = 0$ f' DNE

$-4x = 0$ never

$x = 0$



relative MAX $(0, \underset{f(0)=0+3}{3})$

no relative min (A)

(23) $f(x) = -2x^2 - 12x - 18$ $[-4, -2]$

$f'(x) = -4x - 12$

crit pts

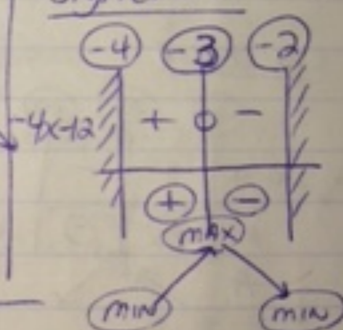
$f'(x) = 0$ f' DNE never   included end pts

$-4x - 12 = 0$ $x = -4$

$-4x = 12$ $x = -2$

$x = -3$

sign chart



Abs MAX $(-3, \underline{0})$ ← $-2(9) - 12(-3) - 18 = -18 + 36 - 18$

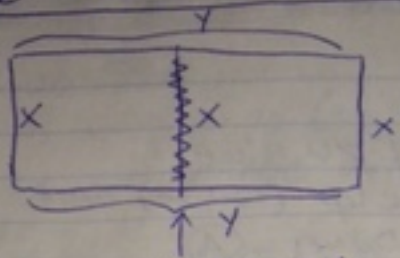
Abs MIN $(-4, \underline{-2})$ ← $-2(16) - 12(-4) - 18 = -32 + 48 - 18$
$= -32 + 30$

Abs MIN $(-2, \underline{-2})$ ← $-2(4) - 12(-2) - 18 = -8 + 24 - 18 = -2$

(both Abs min b/c $-2 = -2$) (A)

CHAB midterm review solns

(24)



ADJACENT side

$P = 400 = 3x + 2y$

maximize

$A = xy$

MAIN EQ

$A = xy$

AUX EQ

$400 = 3x + 2y$

$y = \frac{400 - 3x}{2} = 200 - \frac{3}{2}x$

$A = x\left(200 - \frac{3}{2}x\right) = 200x - \frac{3}{2}x^2$

$A' = 200 - 3x$

CRIT pts:

$A' = 0$

$200 - 3x = 0$

$\boxed{x = \frac{200}{3}}$

KINDA HAS to be the ans b/c NO other CRIT PTS exist

A' DNE never

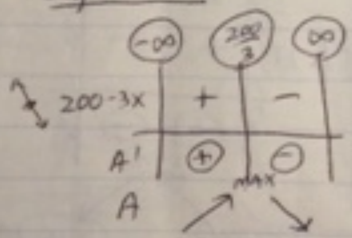
included

end pts – NONE

b/c $x > 0$ NOT included

$y > 0$ not included

sign chart



so max is when adjacent sides are $\frac{200}{3}$ FT

$y = \frac{400 - 3\left(\frac{200}{3}\right)}{2} = \frac{400 - 200}{2}$

$y = 100$ FT ← non-ADJACENT whole sides

so side of pen is $\frac{1}{2}(100)$

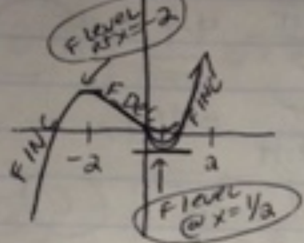
$= \boxed{50\text{FT}}$

D

CHAB - midterm review solns

(25) given $F(x)$, graph $f'(x)$ (the slope of $f(x)$)

$F(x)$ (given)



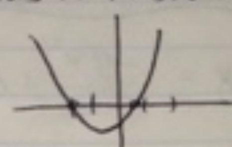
so $F'(x)$ has the following qualities:

① Equal to zero at $x=-2$, $x=1/2$

② positive $(-\infty,-2)\cup(\frac{1}{2},\infty)$

③ Negative $(-2, 1/2)$

thus it has to be (A)

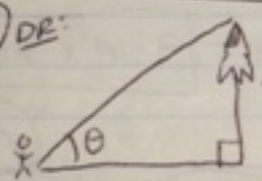


(26) $s(T)=T^3-23T^2+120T$

$s'(T)=v(T)=3T^2-46T+120$ (C)

$s''(T)=v'(T)=A(T)=6T-46$

(27) DE:



E: $\tan\theta=\frac{y}{700}$

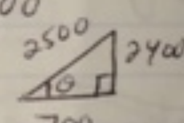
$700\tan\theta=y$

D: $700\sec^2\theta\frac{d\theta}{dT}=\frac{dy}{dT}$

$\frac{d\theta}{dT}=\frac{dy/dT}{700\sec^2\theta}=\frac{(\frac{dy}{dT})}{700}\cos^2\theta$

S: AT moment when $y=2400$

7-24-25 pythagorean triple



$\cos\theta=\frac{700}{2500}=\frac{7}{25}$

$\frac{d\theta}{dt}=\frac{(200)}{700}\left(\frac{7}{25}\right)^2$

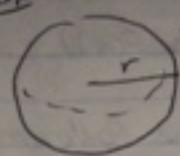
$=\frac{2}{\cancel{7}}\cdot\frac{\cancel{7}}{25}\cdot\frac{7}{25}$

$=\boxed{\frac{14}{625}\text{ rad/sec}}$

(A)

CHAB - midterm review sol/NS

(28) DR:



$\frac{dV}{dT} = \frac{32\pi}{3}\ cm^3/sec$

WANT: $\frac{dr}{dT}$ when $r = 3cm$

E: $V = \frac{4}{3}\pi r^3$

D: $\frac{dV}{dT} = 4\pi r^2 \frac{dr}{dT}$

S: $\frac{32\pi}{3} = 4\pi(3)^2 \frac{dr}{dT}$

$\frac{32\pi}{3} = 4 \cdot 9\pi \frac{dr}{dT}$

$\frac{\overset{8}{\cancel{32}}\cancel{\pi}}{3 \cdot \cancel{4} \cdot 9 \cdot \cancel{\pi}} = dr/dT$

$= \boxed{\frac{8}{27} \frac{cm}{sec}}$ (C)

(29) $\int -4x^3 dx = \frac{-4x^4}{4} + C = \boxed{-x^4 + C}$ (B)

(30) $\int \frac{5}{x} dx = 5\int \frac{1}{x} dx = \boxed{5\ln|x| + C}$ (D)

(31) $\int 3\cos x\, dx = 3\int \cos x\, dx = \boxed{3\sin x + C}$ (A)

(32) $y = -x^2 - 2x + 10$ $[-4, 0]$ use LRAM $n = 4$

$h = \frac{0-(-4)}{4} = \frac{4}{4} = 1 \leftarrow \frac{b-a}{n}$

x	-4	-3	-2	-1	0
y	2	7	10	11	10
	$-16+8+10$	$-9+6+10$	$-4+4+10$	$-1+2+10$	$0+0+10$

LRAM so right endpt never gets used

h·b h·b h·b h·b

$A \approx (1)(2) + (1)(7) + (1)(10) + (1)(11)$

$\approx 2 + 7 + 10 + 11 = \boxed{30}$ (D)

CHAB - midterm review solns

(33) $\int -\frac{4}{x}\,dx = -4\int \frac{1}{x}\,dx = \boxed{-4\ln|x|+C}$

(A)

(34) $\int \frac{1}{\sec x}\,dx = \int \cos x\,dx = \boxed{\sin x + C}$

(C)

(35) FIND TANgent line EQ @ POINT

$y = (x+2)^{1/2} \quad (2,2)$

$y' = \frac{1}{2}(x+2)^{-1/2}(1) = \frac{1}{2\sqrt{x+2}}$

$m_T = y'(2) = \frac{1}{2\sqrt{4}} = \frac{1}{2\cdot 2} = \frac{1}{4}$ ← so either A or B

$y-2 = \frac{1}{4}(x-2)$

$y-2 = \frac{1}{4}x - \frac{1}{2}$

$y = \frac{1}{4}x - \frac{1}{2} + 2 = \boxed{\frac{1}{4}x + \frac{3}{2}}$ (A)

CHAB-midterm review sol/NS

(3b) $f(x) = -2x^3 - 2x^2$

STUFF we CAN FIND from original fxn (w/o calculus)

① Y-INT (x=0)

$y = -2(0^3) - 2(0^2)$

$\boxed{y=0}$

② X-INT (Y=0)

$0 = -2x^3 - 2x^2$

$0 = -2x^2(x+1)$

$0 = -2x^2 \quad x+1=0$

$\boxed{x=0} \quad \boxed{x=-1}$

STUFF we NEED to use the first Derivative for...

$f'(x) = -6x^2 - 4x$

③ CRITICAL POINTS

Ⓐ $f'(x) = 0$

$0 = -6x^2 - 4x$

$0 = -2x(3x+2)$

$-2x = 0 \quad 3x+2=0$

$\boxed{x=0 \quad x=-2/3}$ ← x-coordinates of All CRITICAL PTS

Ⓑ f' DNE

never

(polynomial)

Ⓒ included end pts

none

④ INCREASING / DECREASING / EXTREMA – 1st Derivative Sign Chart

	$-\infty$		$-2/3$		0		∞
$-2x$		+		+	0	−	
$3x+2$		−	0	+		+	
y'		⊖		⊕		⊖	
y		Dec	min	INC	MAX	Dec	

INC $\left(-\frac{2}{3}, 0\right)$

Dec $\left(-\infty, -\frac{2}{3}\right) \cup (0, \infty)$

relative MAX $(0, \underline{0})$

relative MIN $\left(-\frac{2}{3}, \underline{-8/27}\right)$

$y = -2(0) - 2(0) = 0$

$y = -2\left(\frac{-2}{3}\right)^3 - 2\left(\frac{-2}{3}\right)^2$

$= -2\left(\frac{-8}{27}\right) - 2\left(\frac{4}{9}\right)$

$= \frac{16}{27} - \frac{8(3)}{9(3)}$

$= \frac{16}{27} - \frac{24}{27}$

$= -8/27$

CH4b-midterm review solns

36 (continued)

stuff we need to get from the 2nd derivative

$f(x) = -2x^3 - 2x^2$

$f'(x) = -6x^2 - 4x$

$f''(x) = -12x - 4$

⑤ Key points (possible points of inflection)

Ⓐ $f''=0$	f'' DNE	included end pts
$-12x-4=0$	never (polynomial)	(none)
$-12x=4$	
$x=-1/3$	

⑥ concavity/POI from 2nd derivative sign chart

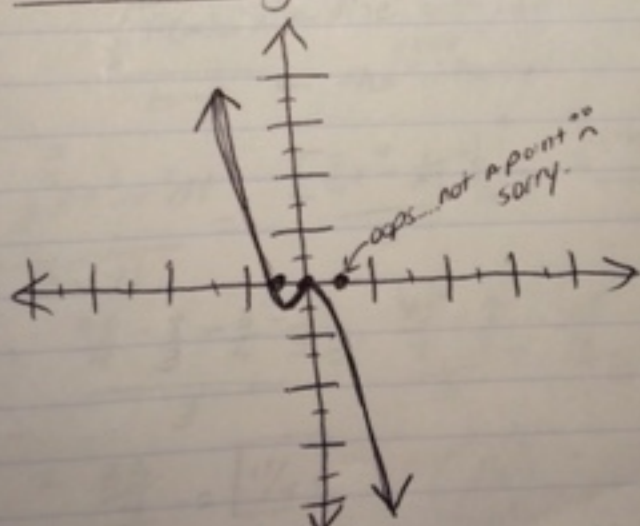
	$-\infty$		$-\frac{1}{3}$		∞
$-12x-4$		+	0	−	
y''		⊕		⊖	
y		∪	POI	∩	

$\boxed{\begin{array}{l}\text{concave up } (-\infty, -\tfrac{1}{3}) \\ \text{concave down } (-\tfrac{1}{3}, \infty)\end{array}}$

x-coordinate of point of inflection

$\boxed{x = -1/3}$

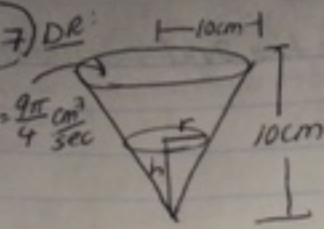
Sketch the graph



CHAB - midterm review solns

(37) DR:

$\frac{dV}{dT} = \frac{9\pi}{4} \frac{cm^3}{sec}$



WANT $\frac{dh}{dT}$ when $h = 6cm$

E: $V = \frac{1}{3}\pi r^2 h$

AUX EQ

$\frac{h}{r} = \frac{10}{10}$

$h = r$

E: $V = \frac{1}{3}\pi h^2 h = \frac{1}{3}\pi h^3$

D: $\frac{dV}{dT} = \pi h^2 \, dh/dT$

S: $\frac{9\pi}{4} = \pi(36)\frac{dh}{dT}$

$\frac{9\pi}{4 \cdot 36\pi} = \boxed{\frac{1}{16}\frac{cm}{sec} = \frac{dh}{dT}}$

4

A: water level is rising at 1/16 cm/sec