

HAAPC - ch. 5 (PC) review ANS (P.1)

$$\textcircled{1} \quad \frac{\csc\theta \cot\theta}{\sec\theta} = \frac{\frac{1}{\sin\theta} \cdot \frac{\cos\theta}{\sin\theta}}{\frac{1}{\cos\theta}} = \frac{\cos\theta}{\sin^2\theta}, \frac{\cos\theta}{1} = \frac{\cos^2\theta}{\sin^2\theta}$$
$$= \boxed{\cot^2\theta} \quad \textcircled{D}$$

$$\textcircled{2} \quad \frac{\tan\theta}{\cot\theta} = \frac{\tan\theta}{\left(\frac{1}{\tan\theta}\right)} = \tan\theta \cdot \frac{\tan\theta}{1} = \boxed{\tan^2\theta} \quad \textcircled{D}$$

$$\textcircled{3} \quad \underline{\sin^2\theta + \tan^2\theta + \cos^2\theta} = \tan^2\theta + (\sin^2\theta + \cos^2\theta)$$
$$= \tan^2\theta + 1 = \boxed{\sec^2\theta} \quad \textcircled{D}$$

$$\textcircled{4} \quad \frac{\tan\theta}{\sec\theta} = \frac{\left(\frac{\sin\theta}{\cos\theta}\right)}{\left(\frac{1}{\cos\theta}\right)} = \frac{\sin\theta}{\cos\theta} \cdot \frac{\cos\theta}{1} = \boxed{\sin\theta} \quad \textcircled{C}$$

$$\textcircled{5} \quad \sin\theta \cos\theta \sec\theta \csc\theta = \sin\theta \cos\theta \cdot \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta}$$
$$= \frac{\sin\theta \cos\theta}{\sin\theta \cos\theta} = \boxed{1} \quad \textcircled{C}$$

$$\textcircled{6} \quad \cot x \tan x = \frac{1}{\tan x} \cdot \tan x = \boxed{1} \quad \textcircled{A}$$

$$\textcircled{7} \quad \sec(-x) \cos(-x) = \sec x \cdot \cos x = \frac{1}{\cos x} \cos x = \boxed{1}$$
$$B$$

$$\textcircled{8} \quad \cos\left(\frac{\pi}{2} - x\right) \csc(-x) = \sin(x) \cdot -\csc x =$$
$$\sin x \cdot -\frac{1}{\sin x} = \boxed{-1} \quad \textcircled{D}$$

$$\textcircled{9} \quad \frac{\cos\left(\frac{\pi}{2} - x\right) \tan x}{\sin^2 x} = \frac{\sin x \tan x}{\sin^2 x} = \frac{\tan x}{\sin x}$$

$$= \frac{\left(\frac{\sin x}{\cos x}\right)}{\sin x} = \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} = \frac{1}{\cos x} = \boxed{\sec x} \quad \textcircled{A}$$

HA2PC - ch. 5 (PC) Ans - review p.2

10 $(\sin^2 x + \cos^2 x) - (\csc^2 x - \cot^2 x) = 1 - 1 = \boxed{0}$

IDS:
 $\sin^2 \theta + \cos^2 \theta = 1$ $\cot^2 \theta + 1 = \csc^2 \theta$
 $1 = \csc^2 \theta - \cot^2 \theta$

(D)

11 $\frac{1 - \sin^2 x}{\sin x - \csc x} = \frac{1 - \sin^2 x}{\frac{\sin x}{1} - \frac{1}{\sin x}} = \frac{1 - \sin^2 x}{\frac{\sin^2 x - 1}{\sin x}}$ (B)
 $= 1 - \sin^2 x \cdot \frac{\sin x}{\sin^2 x - 1} = \frac{(1 - \sin^2 x) \cdot \sin x}{-1(1 - \sin^2 x)} = \boxed{-\sin x}$

12 $\cos x + \sin x \tan x = \cos x + \sin x \frac{\sin x}{\cos x}$
 $= \frac{\cos x}{1} + \frac{\sin^2 x}{\cos x} = \frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos x} =$
 $\frac{\cos^2 x + \sin^2 x}{\cos x} = \frac{1}{\cos x} = \boxed{\sec x}$ (D)

13 $(\csc y + \cot y)(\csc y - \cot y)$ FOIL!!
 $\csc y$

$$= \csc^2 y + \csc y \cot y - \csc y \cot y - \cot^2 y$$

$$= \frac{\csc^2 y - \cot^2 y}{\csc y}$$

$$= \frac{1}{\csc y} = \boxed{\sin y}$$
 (B)

IDS:
 $\cot^2 \theta + 1 = \csc^2 \theta$
 $1 = \csc^2 \theta - \cot^2 \theta$

HA2PC - ch. 5 (PC) review Ans - p.3

$$\begin{aligned}
 14) \frac{\cot x}{\sec^2 x} + \frac{\cot x}{\csc^2 x} &= \frac{\cos x}{\sin x} + \frac{\cos x}{\sin x} \\
 &\left(\frac{1}{\cos^2 x} \right) + \left(\frac{1}{\sin^2 x} \right) \\
 &= \frac{\cos x}{\sin x} \cdot \frac{\cos^2 x}{1} + \frac{\cos x}{\sin x} \cdot \frac{\sin^2 x}{1} = \frac{\cos^3 x + \cos x \sin^2 x}{\sin x} \\
 &= \frac{\cos x (\cos^2 x + \sin^2 x)}{\sin x} = \frac{\cos x (1)}{\sin x} = \frac{\cos x}{\sin x} = \boxed{\cot x} \quad (C)
 \end{aligned}$$

$$\begin{aligned}
 15) \frac{1}{1-\cos x} + \frac{1}{1+\cos x} &\text{ get common denominator...} \\
 &\frac{(1+\cos x)}{(1+\cos x)} \left(\frac{1}{1-\cos x} \right) + \frac{1}{1+\cos x} \frac{(1-\cos x)}{(1-\cos x)} \\
 &= \frac{1+\cos x + 1-\cos x}{1+\cos x - \cos x - \cos^2 x} = \frac{2}{1-\cos^2 x} = \frac{2}{\sin^2 x} \\
 &= \boxed{2 \csc^2 x} \quad (B)
 \end{aligned}$$

$$16) \csc^2 x - 1 = (\csc x)^2 - 1^2 = \boxed{(\csc x + 1)(\csc x - 1)} \quad (B)$$

$$17) 4 \cot^2 x - \frac{4}{\tan x} + \cos x \sec x$$

$$= 4 \cot^2 x - 4 \cot x + \cos x \cdot \frac{1}{\cos x}$$

$$= 4 \cot^2 x - 4 \cot x + 1 \quad \text{let } u = \cot x$$

$$= 4u^2 - 4u + 1$$

$$= (2u-1)(2u-1)$$

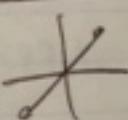
$$= (2 \cot x - 1)(2 \cot x - 1) = (2 \cot x - 1)^2$$

(C)

HA2PC - ch.5 (PC) review Ans - p. 4

(18) $1 - 2\sin^2 x + \sin^4 x$
 $= \sin^4 x - 2\sin^2 x + 1$ let $u = \sin^2 x$
 $= u^2 - 2u + 1$
 $= (u-1)(u-1) = (\sin^2 x - 1)(\sin^2 x - 1)$
 DIFF. OF SQS!! DIFF. OF SQS!!
 $= (\sin x + 1)(\sin x - 1)(\sin x + 1)(\sin x - 1)$
 right but... not what they wanted... see answer choices
 $= (\sin^2 x - 1)(\sin^2 x - 1)$ ID: $\sin^2 \theta + \cos^2 \theta = 1$
 $= (-\cos^2 x)(-\cos^2 x)$ $\sin^2 \theta - 1 = -\cos^2 \theta$
 $= \boxed{\cos^4 x}$ A

(19) $1 - \sin^3 x$ sum/diff of cubes!!
 $1^3 - (\sin x)^3$ $A^3 \pm B^3 = (A \pm B)(A^2 \mp AB + B^2)$
 "keep the sign, change the sign, always a positive"
 $= \boxed{(1 - \sin x)(1^2 + \sin x + \sin^2 x)}$ C

(20) $\cos x = \sin x$ 
 $\frac{x}{x - \pi n} = \frac{y}{y - \pi n}$ $x = \frac{\pi}{4}, \frac{5\pi}{4}$ A

(21) $\sec^2 x - 2 = \tan^2 x$ ID: $\sec^2 \theta = 1 + \tan^2 \theta$
 $1 + \tan^2 x - 3 = \tan^2 x$
 $-2 = 0$ lie NO SOLN B

MA2PC - ch.5 (PC) review solns p.5

(22) $\sin^2 x + \sin x = 0$

$$\sin x (\sin x + 1) = 0$$

$$\sin x = 0$$

$$\sin x + 1 = 0$$

$$\sin x = -1$$

$$x \neq$$

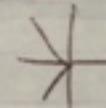
$$x = 0, \pi$$

$$+$$

$$x = \frac{3\pi}{2}$$

(B)

(23) $\cos x = -\frac{1}{2}$



(D)

$$x = \frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi$$

(24) $\cos^2 x + 2\cos x + 1 = 0$

let
 $u = \cos x$

$$u^2 + 2u + 1 = 0$$

$$(u+1)(u+1) = 0$$

$$(\cos x + 1)(\cos x + 1) = 0$$

$$\begin{array}{l} \cos x + 1 = 0 \\ \cos x = -1 \\ x \neq \end{array}$$

$$x = \pi + 2n\pi$$

(A)

(25) $\sin 2x = -\sin x$

$$2\sin x \cos x = -\sin x$$

$$2\sin x \cos x + \sin x = 0$$

$$\sin x (2\cos x + 1) = 0$$

$$\sin x = 0$$

$$2\cos x + 1 = 0$$

$$\cos x = -\frac{1}{2}$$

$$x \neq$$

$$x = 0, \pi$$

$$+$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

(D)

(A)

MA2PC-ch.5(PC) review ANS p. 6

(26) $2\cos x + \sin 2x = 0$

$$2\cos x + 2\sin x \cos x = 0$$

$$2\cos x (1 + \sin x) = 0$$

$$2\cos x = 0 \quad 1 + \sin x = 0$$

$$\cos x = 0$$

$$1 + \sin x = 0$$

\cancel{x}

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

\cancel{x}

$$x = \frac{3\pi}{2}$$

(27) $\cos 4x - \cos 2x = 0$

$$\cos 2\theta = 2\cos^2 \theta - 1$$

\downarrow

$$2\cos^2(2x) - 1 - \cos 2x = 0$$

$$2\cos^2(2x) - \cos 2x - 1 = 0 \quad \text{let } u = \cos 2x$$

$$2u^2 - u - 1 = 0$$

$$(2u+1)(u-1) = 0$$

$$2\cos 2x + 1 = 0 \quad \cos 2x - 1 = 0$$

$$\cos 2x = -\frac{1}{2}$$

$$\cos 2x = 1$$

\cancel{x} $\overset{+6\pi}{\cancel{x}}$

$$2x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$$

\cancel{x} $\overset{+2\pi}{\cancel{x}}$

$$2x = 0, 2\pi$$

go around
u.c.
twice b/c
"2x"
inside cos

$$x = \frac{2\pi}{6}, \frac{4\pi}{6}, \frac{8\pi}{6}, \frac{10\pi}{6} \quad x = \frac{0, 2\pi}{2}$$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, 0, \pi$$

(C)

MA2PC - ch.5(PC) review Ans p.7

(28) $\sin 2x - \cos 2x$

$2\sin x \cos x$

↑ look at ans choices
to see which
of three options
to choose... All ans
have $\sin^2 x$
but NO
 $\cos^2 x$...

so choose

$$\cos 2\theta = 1 - 2\sin^2 \theta$$

$$\sin 2x - \cos 2x = 2\sin x \cos x - (1 - 2\sin^2 x)$$

$$= 2\sin x \cos x - 1 + 2\sin^2 x$$

$$= \boxed{2\sin^2 x + 2\sin x (\cos x - 1)}$$

(C)

(29) $\cos 2x - \sin x$

↑
look at
ans to figure which of the formulae to use
All ans have both
 $\cos^2 x$ and $\sin^2 x$...
so use

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\cos 2x - \sin x = \boxed{\cos^2 x - \sin^2 x - \sin x}$$

(D)

BONNIE

$$\cos 4x - \cos 2x = 0$$

A) $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

B) 0, $\frac{2\pi}{3}, \frac{4\pi}{3}$

C) 0, $\frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$

D) No solution

27)

Rewrite with only $\sin x$ and $\cos x$.

28) $\sin 2x - \cos 2x$

A) $2 \sin^2 x - 2 \sin x \cos x - 1$

B) $2 \sin x$

C) $2 \sin^2 x + 2 \sin x \cos x - 1$

D) $2 \sin^2 x - 2 \sin x \cos x + 1$

28)

29) $\cos 2x - \sin x$

A) $\cos^2 x - \sin^3 x$

B) $\cos^2 x + \sin^2 x + \sin x$

C) $\cos^2 x - 3 \sin x$

D) $\cos^2 x - \sin^2 x - \sin x$

29)

30) $\sin 2x - \cos 3x$

A) $2 \sin x \cos x + \cos x - 4 \cos x \sin 2x$

B) $3 \sin^2 x \cos x - \sin 3x + 2 \sin x \cos x$

C) $\cos^3 x + 2 \sin^2 x \cos x - \sin^2 x + 2 \sin x \cos x$

D) $2 \sin^2 x \cos x - \cos^3 x - 2 \sin x \cos x$

Ans sheet says "A"
but I think it's A
TYPO... I think A is
correct but should be option A
the Ans I got for option A

30)

(30) $\sin 2x - \cos 3x = 2 \sin x \cos x - \cos 3x = 2 \sin x \cos x - \cos(x+2x)$

$$= 2 \sin x \cos x - [\cos x \cos 2x - \sin x \sin 2x]$$
$$= 2 \sin x \cos x - \cos x \cos 2x + \sin x (2 \sin x \cos x)$$
$$= 2 \sin x \cos x - \cos x \underline{\cos 2x} + 2 \sin^2 x \cos x$$

it all depends on which of the three choices I plug in for $\cos 2x$...

OPT 1: $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$

gives me ...
 $2 \sin x \cos x - \cos x (\cos^2 x - \sin^2 x) + 2 \sin^2 x \cos x$

$$= 2 \sin x \cos x - \cos^3 x + \sin^2 x \cos x + 2 \sin^2 x \cos x = [2 \sin x \cos x - \cos^3 x + 3 \sin^2 x \cos x]$$

OPT 2: $\cos 2\theta = 1 - 2 \sin^2 \theta$

gives me ...
 $2 \sin x \cos x - \cos x (1 - 2 \sin^2 x) + 2 \sin^2 x \cos x = 2 \sin x \cos x - \cos x + 4 \sin^2 x \cos x$

$$= [2 \sin x \cos x - \cos x + 4 \sin^2 x \cos x]$$

Not a choice

OPT 3: $\cos 2\theta = 2 \cos^2 \theta - 1$

gives me ...
 $2 \sin x \cos x - \cos x (2 \cos^2 x - 1) + 2 \sin^2 x \cos x = [2 \sin x \cos x - 2 \cos^3 x + \cos x + 2 \sin^2 x \cos x]$

WHAT IF I take option 3 AND replace $\cos^2 x$ with $1 - \sin^2 x$ (not sure if it'll work)

$$= 2 \sin x \cos x - 2 \cos x (\cos^2 x) + \cos x + 2 \sin^2 x \cos x$$

$$= 2 \sin x \cos x - 2 \cos x (1 - \sin^2 x) + \cos x + 2 \sin^2 x \cos x$$

$$= 2 \sin x \cos x - \cos x + 2 \sin^2 x \cos x + 2 \sin^2 x \cos x = [2 \sin x \cos x - \cos x + 4 \sin^2 x \cos x]$$

also not a choice

very similar to "A" but not A

choice

SAME AS option #2

~~SECRET~~

$$\sin 2x - \cos 3x$$

PICK $x = \frac{\pi}{6}$ (b/c 6 is multiple of both 2 and 3. so $2(\frac{\pi}{6})$ AND $3(\frac{\pi}{6})$ are both ON U.C.)

$$\text{IF } x = \frac{\pi}{6}$$

$$\sin(2 \cdot \frac{\pi}{6}) - \cos(3 \cdot \frac{\pi}{6}) = \sin \frac{\pi}{3} - \cos \frac{\pi}{2} = \frac{\sqrt{3}}{2} - 0 = \boxed{\frac{\sqrt{3}}{2}}$$

Now check which answer(s) give us $\frac{\sqrt{3}}{2}$ IF $x = \frac{\pi}{6}$

(A) $2\sin(\frac{\pi}{6})\cos(\frac{\pi}{6}) + \cos(\frac{\pi}{6}) - 4\cos(\frac{\pi}{6})\sin^2(\frac{\pi}{6})$

$$= 2(\frac{1}{2})(\frac{\sqrt{3}}{2}) + \frac{\sqrt{3}}{2} - 4(\frac{\sqrt{3}}{2})(\frac{1}{2})^2 = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} = \boxed{\frac{\sqrt{3}}{2}} \checkmark$$

(B) $3\sin^2(\frac{\pi}{6})\cos(\frac{\pi}{6}) - \sin^3(\frac{\pi}{6}) + 2\sin(\frac{\pi}{6})\cos(\frac{\pi}{6})$
 $3(\frac{1}{2})^2 \frac{\sqrt{3}}{2} - (\frac{1}{2})^3 + 2(\frac{1}{2})\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{8} - \frac{1}{8} + \frac{\sqrt{3}}{8} = \frac{4\sqrt{3}}{8} - \frac{1}{8} = \boxed{\frac{4\sqrt{3}-1}{8}}$

(C) $\cos^3(\frac{\pi}{6}) + 2\sin^2(\frac{\pi}{6})\cos(\frac{\pi}{6}) - \sin^2(\frac{\pi}{6}) + 2\sin(\frac{\pi}{6})\cos(\frac{\pi}{6})$

$$= (\frac{\sqrt{3}}{2})^3 + 2(\frac{1}{4})(\frac{\sqrt{3}}{2}) - (\frac{1}{4}) + 2(\frac{1}{2})(\frac{\sqrt{3}}{2}) = \frac{3\sqrt{3}}{8} + \frac{\sqrt{3}}{4} - \frac{1}{4} + \frac{\sqrt{3}}{8}$$

 $= \frac{3\sqrt{3}}{8} + \frac{2\sqrt{3}}{8} + \frac{\sqrt{3}}{8} - \frac{2}{8} = \boxed{\frac{6\sqrt{3}-2}{8}}$

(D) $2\sin^2(\frac{\pi}{6})\cos(\frac{\pi}{6}) - \cos^3(\frac{\pi}{6}) - 2\sin(\frac{\pi}{6})\cos(\frac{\pi}{6})$

$$= 2(\frac{1}{4})(\frac{\sqrt{3}}{2}) - (\frac{\sqrt{3}}{2})^3 - 2(\frac{1}{2})(\frac{\sqrt{3}}{2}) = \frac{\sqrt{3}}{4} - \frac{3\sqrt{3}}{8} - \frac{\sqrt{3}}{2} = \frac{2\sqrt{3}}{8} - \frac{3\sqrt{3}}{8} - \frac{4\sqrt{3}}{8}$$

 $= \boxed{\frac{5\sqrt{3}}{8}} \text{ NO}$

SO IT MUST be A. ← but I can't figure out a mathy way to get there :)

{WQ: |||HF.

-Jz z3■

DA fEBRQ|||3>|||GUGXNSMDLg-eE|||PAGf Vd

r>LSS+e ■

This review is identical in format to the Ch.5 Exam. Only the actual numerical values and particular equations will vary.

Multiple-Choice:

Record all answers to the multiple-choice questions here. To clearly distinguish between A and D, it is recommended that you use capital letters. (2 points each)

Free-Response: (40 points...points are listed in *italics* next to each problem)

You must show a reasonable amount of work that leads to your answer. Where it is impossible to show your work, explain the mental leaps that you made to draw your conclusion. Where estimation is required, round or truncate all answer to the thousandths place.

31. Solve $\sqrt{3}\csc x - 2 = 0$ for all possible values of x in the interval $[0, 2\pi]$. (5 points)

$$\sqrt{3}\csc x = 2 \quad \rightarrow \sin x = \frac{\sqrt{3}}{2}$$

$$\csc x = \frac{2}{\sqrt{3}}$$

↙

$$X = \frac{\pi}{3}, \frac{2\pi}{3}$$

32. Solve $\tan 3x(\tan x - 1) = 0$ for all possible values of x in the interval $[0, 2\pi]$.

(8 points) $\tan 3x(\tan x - 1) = 0$

$$\begin{cases} \tan 3x = 0 \\ \tan x - 1 = 0 \end{cases}$$

$$\begin{cases} 3x = 0, \pi, 2\pi, 3\pi, 4\pi \\ \tan x = 1 \end{cases}$$

↙

$$x = \frac{\pi}{7}, \frac{5\pi}{7}$$

ANS:

$$x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{\pi}{4}, \frac{5\pi}{4}$$

33. Solve $\tan^2 x - \sec x = 1$ for all possible values of x in the interval $[0, 2\pi]$.

(9 points) $\tan^2 x - \sec x = 1$

$$\begin{aligned} \text{ID: } \sec^2 \theta &= \tan^2 \theta + 1 \\ \sec^2 \theta - 1 &= \tan^2 \theta \\ \sec^2 x - 1 - \sec x &= 1 \\ \sec^2 x - \sec x - 2 &= 0 \\ \text{let } \sec x = u & \\ u^2 - u - 2 &= 0 \end{aligned}$$

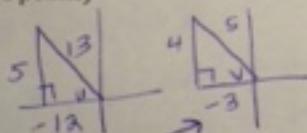
$$\begin{aligned} (u-2)(u+1) &= 0 \\ (\sec x - 2)(\sec x + 1) &= 0 \\ \sec x = 2 & \quad \sec x + 1 = 0 \\ \cos x = \frac{1}{2} & \quad \sec x = -1 \\ \cos x = \pm \frac{1}{2} & \end{aligned}$$

↙

$$x = \frac{\pi}{3}, \frac{5\pi}{3}, \pi$$

34. Given $\sin u = \frac{5}{13}$, $\cos v = -\frac{3}{5}$ and that both u and v lie in Quadrant II. Find $\sin(u-v)$.

(8 points)



$$\begin{aligned} \sin(u-v) &= \sin u \cos v - \cos u \sin v \\ &= \frac{5}{13} \left(-\frac{3}{5}\right) - \left(\frac{12}{13}\right)\left(\frac{4}{5}\right) = -\frac{15}{65} + \frac{48}{65} = \frac{33}{65} \end{aligned}$$

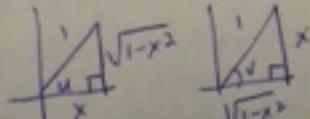
$$\boxed{\frac{33}{65}}$$

35. Rewrite $\cos(\arccos x + \arcsin x)$ as an algebraic expression (no trigonometric functions.) (10 points)

$$\cos(\arccos x + \arcsin x) = \cos(u+v) = \cos u \cos v - \sin u \sin v = \frac{x}{1} \sqrt{1-x^2} - \frac{\sqrt{1-x^2}}{1} \cdot \frac{x}{1}$$

$$= x \sqrt{1-x^2} - x \sqrt{1-x^2}$$

$$\begin{aligned} \arccos x &= u \\ x &= \cos u \\ \frac{x}{1} &= \sin v \\ \frac{\sqrt{1-x^2}}{1} &= \cos v \end{aligned}$$



$$= 0$$