

## POLAR UNIT

$$\textcircled{1} \quad r = 2 - 3\sin\theta \quad \text{find slope } \left(\frac{dy}{dx}\right) \text{ at } (2, \pi)$$

$$x = r\cos\theta = (2 - 3\sin\theta)\cos\theta$$

$$y = r\sin\theta = (2 - 3\sin\theta)\sin\theta$$

$$\frac{dx}{d\theta} = (2 - 3\sin\theta)(-\sin\theta) + \cos\theta(-3\cos\theta)$$

$$\boxed{\frac{dx}{d\theta} = -2\sin\theta + 3\sin^2\theta - 3\cos^2\theta} \quad \text{← Denominator}$$

$$\frac{dy}{d\theta} = (2 - 3\sin\theta)(\cos\theta) + \sin\theta(-3\cos\theta)$$

$$\frac{dy}{d\theta} = 2\cos\theta - 3\sin\theta\cos\theta - 3\sin\theta\cos\theta$$

$$\boxed{\frac{dy}{d\theta} = 2\cos\theta - 6\sin\theta\cos\theta} \quad \text{← Numerator}$$

$$\frac{dy/d\theta}{dx/d\theta} = \frac{dy}{dx} = \frac{(2\cos\theta - 6\sin\theta\cos\theta)}{(-2\sin\theta + 3\sin^2\theta - 3\cos^2\theta)}$$

at  $(2, \pi)$

$$\left. \frac{dy}{dx} \right|_{(2, \pi)} = \frac{2\cos\pi - 6\sin\pi\cos\pi}{-2\sin\pi + 3\sin^2\pi - 3\cos^2\pi} = \frac{2(-1) + \phi}{\phi + \phi - 3(-1)^2} = \frac{-2}{-3}$$

$$\boxed{\left. \frac{dy}{dx} \right|_{(2, \pi)} = \frac{2}{3}}$$

$$\textcircled{2} \quad r = 3\sin(2\theta) \quad \text{at} \quad \theta = \frac{\pi}{3}$$

For EQ of tangent line, we need  
POINT AND Slope

POINT:  $x = r\cos\theta = 3\sin(2\theta)\cos\theta$   
 $x = 3\sin\left(\frac{2\pi}{3}\right)\cos\frac{\pi}{3} = 3\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$   
 $x = \frac{3\sqrt{3}}{4}$

$$y = r\sin\theta = 3\sin(2\theta)\sin\theta$$

$$y = 3\sin\left(\frac{2\pi}{3}\right)\sin\left(\frac{\pi}{3}\right) = 3\frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2} = \frac{9}{4}$$

POINT is  $\left(\frac{3\sqrt{3}}{4}, \frac{9}{4}\right)$

Slope: Find  $\frac{dy}{dx}$  at POINT...

$$y = 3\sin u \cos v$$

$$\frac{dy}{d\theta} = 3\sin 2\theta \cdot \cos\theta + \sin\theta \cdot 3\cos 2\theta \cdot 2$$

$$x = 3\sin 2\theta \cos\theta$$

$$\frac{dx}{d\theta} = 3\sin 2\theta (-\sin\theta) + \cos\theta \cdot 3\cos(2\theta) \cdot 2$$

$$\text{so } \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{3\sin 2\theta \cos\theta + 6\sin\theta \cos 2\theta}{-3\sin 2\theta \sin\theta + 6\cos\theta \cos 2\theta}$$

$$\left. \frac{dy}{dx} \right|_{\theta=\frac{\pi}{3}} = \frac{3\sin^2 \frac{\pi}{3} \cos \frac{\pi}{3} + 6\sin \frac{\pi}{3} \cos 2\frac{\pi}{3}}{-3\sin^2 \frac{\pi}{3} \sin \frac{\pi}{3} + 6\cos \frac{\pi}{3} \cos 2\frac{\pi}{3}}$$

$$= \frac{3\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) + 6\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right)}{-3\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + 6\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)} = \frac{\frac{3\sqrt{3}}{4} - \frac{6\sqrt{3}}{4}}{-\frac{9}{4} - \frac{6}{4}}$$

$$= \frac{-\frac{3\sqrt{3}}{4}}{-\frac{15}{4}} = -\frac{3\sqrt{3}}{4} \cdot \frac{4}{-15} = \frac{-3\sqrt{3}}{-15} = \boxed{\frac{\sqrt{3}}{5}}$$

② (continued)

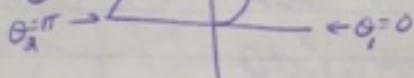
so we have the point  $\left(\frac{3\sqrt{3}}{4}, \frac{9}{4}\right)$   
and slope  $m = \frac{\sqrt{3}}{5}$

now we use point-slope as usual...

$$y - \frac{9}{4} = \frac{\sqrt{3}}{5} \left(x - \frac{3\sqrt{3}}{4}\right)$$

③  $r = \theta$

$$\theta_1 = \pi$$



Area of a polar curve is  $A = \int_{\theta_1}^{\theta_2} \frac{1}{2} r^2 d\theta$

so

$$A = \int_0^{\pi} \frac{1}{2} \theta^2 d\theta$$

④  $r = 1 + \sin \theta$

$$A = \int_{-\pi/2}^{\pi/2} (1 + \sin \theta)^2 d\theta$$



clearly it  
starts and  
ends with  $r=0$

$$\theta = 1 + \sin \theta$$

$$-1 = \sin \theta$$

$$\theta = -\frac{\pi}{2}, \frac{3\pi}{2}, \dots$$

$$\theta_1 = -\frac{\pi}{2}, \theta_2 = \frac{3\pi}{2}$$

\*Note, I disagree w/  
the packet answer  
(it has the wrong bounds)

⑤



$$r = 2 \sin 4\theta$$

Trying to find the area of one leaf... so starting and ending w/  $r=0$  (at the pole)

$$\theta = 2 \sin 4\theta$$

$$\theta = \sin 4\theta$$

$$4\theta = 0, \pi, 2\pi, 3\pi, \dots$$

$$\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \dots$$

so these are the bounds for the 1st leaf

$$A = \int_0^{\frac{\pi}{4}} \frac{1}{2} (2 \sin 4\theta)^2 d\theta$$

NOTE: to view polar graphs in your calculator:

① MODE choose **POL**

② Now **[Y=]** actually gives you **[r=]**

and **[X,T,θ]** gives you **[θ]** ← in fxn mode it give you x

③ You set how much of the polar curve you want to see in **[WINDOW]**

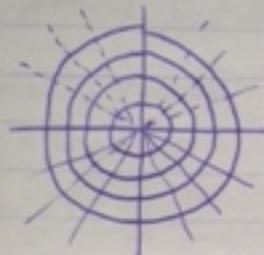
$\theta_{\text{MIN}} = \theta_1$ , (start of curve)  
you want

$\theta_{\text{MAX}} = \theta_2$  (end of curve)  
you want

④ You can still integrate w/ **[MATH] [9]**  
but when you go to **[VARS]** to  
get your fxn go to **[Y-VARS]**  
then **[POLAR]**

⑥ sketch the polar region described by the following integral  
 (NOTE: polar graph paper is not a grid ... it's sets of concentric circles about the pole (origin))

like this: (badly drawn by me)



with lines at the common angles (like unit circle angles)

so on a grid like that, we'd sketch

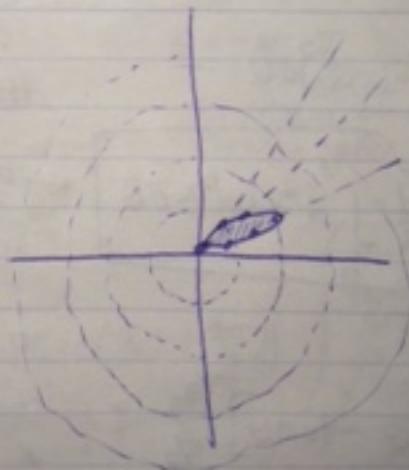
$$r = \sin(3\theta) \quad \text{since } A = \frac{1}{2} \int_{\theta_1}^{\theta_2} r^2 d\theta$$

from  $\theta_1 = 0$   
 to  $\theta_2 = \pi/3$

$$= \frac{1}{2} \int_0^{\pi/3} \sin^2(3\theta) d\theta$$

by hand, we'd have to make a table of easy  $\theta$ 's from 0 to  $\pi/3$   
 (note you will be multiplying them by 3...)

$\theta$	$r = \sin(3\theta)$
0	$\sin(0) = 0$
$\pi/6$	$\sin(\pi/6) = 1/2$
$\pi/4$	$\sin(\pi/4) = \sqrt{2}/2$
$\pi/3$	$\sin(\pi/3) = \sqrt{3}/2$
$\pi/2$	$\sin(\pi/2) = 1$
$2\pi/3$	$\sin(2\pi/3) = \sqrt{3}/2$
$3\pi/4$	$\sin(3\pi/4) = \sqrt{2}/2$
$5\pi/6$	$\sin(5\pi/6) = 1/2$
$\pi$	$\sin(\pi) = 0$



⑦ A In polar coordinates of  
 ① Line  $x=1$  and ②  $x^2+y^2=2^2$

$$x=1$$

$$x=r\cos\theta$$

$$\text{so } 1=r\cos\theta$$

$$\frac{1}{\cos\theta} = r$$

$$\boxed{\sec\theta = r}$$

The answer  
key incorrectly  
says  
 $r=2\sec\theta$

Circle of radius  
2 centered at (0,0)

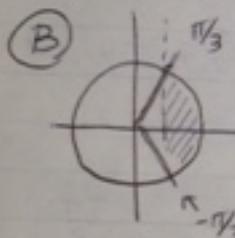
$$\text{recall } x^2+y^2=r^2 \leftarrow \text{to help convert}$$

$$\text{so } x^2+y^2=4=r^2$$

$$\boxed{r=2}$$

A  $\boxed{r=\sec\theta}$  for  $x=1$

$\boxed{r=2}$  for  $x^2+y^2=2^2 \leftarrow \text{circle at (0,0) of } r=2$



$$\text{Area} = \frac{1}{2} \int_{\text{INT}_1}^{\text{INT}_2} \text{outside curve}^2 - \text{inside curve}^2 d\theta$$

FIND bounds:  $\sec\theta = 2$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}, -\frac{\pi}{3} \leftarrow \text{and also lots of other places, but these are the two that make sense}$$

$$A = \frac{1}{2} \int_{-\pi/3}^{\pi/3} (2)^2 - (\sec\theta)^2 d\theta$$

$$\boxed{\text{ANS: } \frac{1}{2} \int_{-\pi/3}^{\pi/3} 4 - \sec^2\theta d\theta}$$

$\leftarrow$  b/c Area of polar curve is  $\frac{1}{2} \int_{\theta_1}^{\theta_2} r^2 d\theta \dots$  if two curves,

$$\text{it's } \frac{1}{2} \int_{\theta_1}^{\theta_2} R^2 - r^2 d\theta$$

$$\textcircled{C} \quad \frac{1}{2} [4\theta - \tan\theta]_{-\pi/3}^{\pi/3}$$

$\tan^{-1}$  odd fxn

$$= \frac{1}{2} \left[ \left( 4\frac{\pi}{3} - \tan\frac{\pi}{3} \right) - \left( -4\frac{\pi}{3} + \tan\left(-\frac{\pi}{3}\right) \right) \right]$$

$$= \frac{1}{2} \left( 4\frac{\pi}{3} - \tan\frac{\pi}{3} + 4\frac{\pi}{3} + \tan\frac{\pi}{3} \right)$$

$$⑦ \textcircled{c} = \frac{1}{2} \left[ 2\left(\frac{\pi}{3}\right) - 2\left(-\tan\frac{\pi}{3}\right) \right]$$

$$= \frac{4\pi}{3} + 2\tan\frac{\pi}{3} = \boxed{\frac{4\pi}{3} - \sqrt{3}} \approx \boxed{2.456} \quad (7)$$

OR...

$$\frac{4\pi}{3} - \sqrt{3} \cdot \frac{3}{3} = \boxed{\frac{4\pi - 3\sqrt{3}}{3}}$$

$$⑧ r^2 = 4 \cos 2\theta \quad r = \sqrt{2}$$

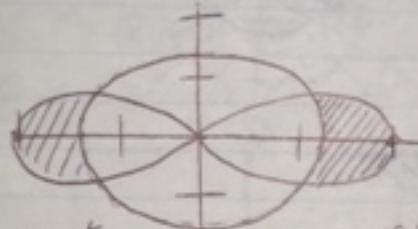
$$r = \pm \sqrt{4 \cos 2\theta}$$

$$r_1 = \sqrt{4 \cos 2\theta}$$

$$r_2 = -\sqrt{4 \cos 2\theta}$$

$$r_3 = \sqrt{2}$$

← IF YOU  
DO THIS ON  
CALC



FIND bounds...

$$r = \sqrt{2}$$

$$r^2 = 2$$

$$r^2 = r^2$$

$$2 = 4 \cos 2\theta$$

$$\frac{1}{2} = \cos 2\theta$$

$$2\theta = \dots, \frac{-5\pi}{3}, \frac{-\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}, \dots$$

$$\theta = \dots, \frac{-5\pi}{6}, \frac{-\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$$

So these bounds  
clearly refer to  $\frac{-\pi}{6}$  to  $\frac{\pi}{6}$

FIND 1st petal A...

$$A = \frac{1}{2} \int_{\theta_1}^{\theta_2} r^2 d\theta = \frac{1}{2} \int_{\theta_1}^{\theta_2} R^2 - r^2 d\theta$$

$$A = \frac{1}{2} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} R^2 - r^2 d\theta$$

$$A = \frac{1}{2} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} 4 \cos 2\theta - 2 d\theta$$

$$A = \frac{1}{2} \left[ 2 \sin 2\theta - 2\theta \right]_{-\frac{\pi}{6}}^{\frac{\pi}{6}}$$

$$A = \frac{1}{2} \left[ (2 \sin \frac{\pi}{3} - \frac{\pi}{3}) - (2 \sin(-\frac{\pi}{3}) + \frac{\pi}{3}) \right]$$

$$A = \frac{1}{2} \left[ 2 \frac{\sqrt{3}}{2} - \frac{\pi}{3} - 2(-\frac{\sqrt{3}}{2}) - \frac{\pi}{3} \right]$$

$$A = \frac{1}{2} \left[ 2\sqrt{3} - \frac{2\pi}{3} \right]$$

but  
there are  
two EQUAL  
AREAS...  
so double  
this

$$A = 2\sqrt{3} - \frac{2\pi}{3}$$

NOTE ON 8A... IF you have to graph this by HAND...

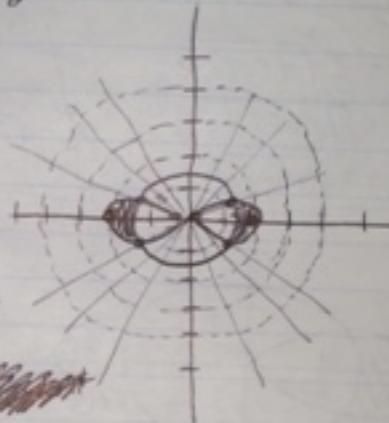
$$r^2 = 4\cos 2\theta \quad \text{and} \quad r = \sqrt{2}$$

$$r = \pm \sqrt{4\cos 2\theta}$$

choose easy  $\theta$   
from  $[0, \pi]$

this is just  
a circle of  $r = \sqrt{2}$   
 $r \approx 1.41$

$\theta$	$\pm \sqrt{4\cos 2\theta}$	$P = \frac{2\pi}{\theta}$
0	$\pm \sqrt{4\cos 0} = \pm \sqrt{4} = \pm 2$	
$\frac{\pi}{6}$	$\pm \sqrt{4\cos \frac{\pi}{3}} = \pm \sqrt{2}$	
$\frac{\pi}{4}$	$\pm \sqrt{4\cos \frac{\pi}{2}} = 0$	
$\frac{\pi}{3}$	$\pm \sqrt{4\cos \frac{2\pi}{3}} = \text{NON REAL}$ ANS	
$\frac{\pi}{2}$	$\pm \sqrt{4\cos \pi} = \text{NON REAL}$ ANS	
$\frac{2\pi}{3}$	$\pm \sqrt{4\cos \frac{4\pi}{3}} = \text{NON REAL}$ ANS	
$\frac{3\pi}{4}$	$\pm \sqrt{4\cos \frac{3\pi}{2}} = 0$	
$\frac{5\pi}{6}$	$\pm \sqrt{4\cos \frac{5\pi}{3}} = \pm \sqrt{2}$	
$\pi$	$\pm \sqrt{4\cos 2\pi} = \pm 2$	



POINTS:

$$(0, 2), (0, -2), (\frac{\pi}{6}, \sqrt{2}), (\frac{\pi}{6}, -\sqrt{2}), (\frac{\pi}{4}, 0)$$

~~$(\frac{3\pi}{4}, 0), (\frac{5\pi}{6}, \sqrt{2}), (\frac{5\pi}{6}, -\sqrt{2}), (\pi, 2), (\pi, -2)$~~

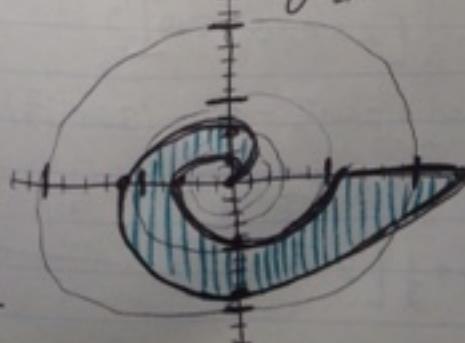
⑨  $r = \theta$   $r = 2\theta$  for  $0 \leq \theta \leq 2\pi$

FIND bounds:

$\theta = 2\theta$   
 $-\theta = -\theta$   
 $0 = \theta$  ← this is the only place they intersect, so the other cut off must be the endpoint

let's graph it...

$r_1 = \theta$	$r_2 = 2\theta$
$0$	$0$
$\frac{\pi}{2}$	$\pi/2 = 1.5$
$\pi$	$\pi/2 = 3$
$\frac{3\pi}{2}$	$3\pi/2 = 9$
$2\pi$	$2(2\pi) = 12$



⑨ (continued)

$$A = \frac{1}{2} \int_0^{2\pi} R^2 - r^2 d\theta$$

$$A = \frac{1}{2} \int_0^{2\pi} (2\theta)^2 - \theta^2 d\theta$$

$$A = \frac{1}{2} \int_0^{2\pi} 4\theta^2 - \theta^2 d\theta = \frac{1}{2} \int_0^{2\pi} 3\theta^2 d\theta$$

$$A = \frac{1}{2} [\theta^3]_0^{2\pi} = \frac{1}{2} [(2\pi)^3 - 0^3] = \frac{8\pi^3}{2}$$

$$\boxed{A = 4\pi^3}$$

⑩  $r = \theta + 2\sin\theta$  for  $0 \leq \theta \leq 2\pi$  \*calculator problem

# POLAR UNIT - AP problems

2013 AP #2 (calculator)

(A) Note there are three pieces

$[0, \frac{\pi}{6}] \rightarrow$  region is inside circle

$[\frac{\pi}{6}, \frac{5\pi}{6}] \rightarrow$  region is inside other curve

$[\frac{5\pi}{6}, 2\pi] \rightarrow$  region is inside circle

$$A = \frac{1}{2} \int_0^{\frac{\pi}{6}} 3^2 d\theta + \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (4 - 2\sin\theta)^2 d\theta + \frac{1}{2} \int_{\frac{5\pi}{6}}^{2\pi} 3^2 d\theta$$

You must have all integrands, limits, and constants in correct math notation to get credit... then, since it's a calculator section, use your calc to integrate... remember you can write calculator syntax on your answer, but it gets no credit...

$$A = \text{FNINT}(.5(9), \theta, 0, \frac{\pi}{6}) +$$

or pull .5  
in front

$$.5 \text{FNINT}((4 - 2\sin(\theta))^2, \theta, \frac{\pi}{6}, \frac{5\pi}{6})$$

easier if you  
make ~~sector~~  $r_1 = 4 - 2\sin\theta$   
then use  $r_1^2$

$$+.5 \text{FNINT}(9, \theta, \frac{5\pi}{6}, 2\pi)$$



$$A \approx 24.708$$

(9)

OR Notice the piece of the circle is missing  $\frac{\pi}{6}$  to  $\frac{5\pi}{6}$ ...  $\frac{\frac{5\pi}{6} - \frac{\pi}{6}}{2\pi} = \frac{2\pi}{3}$

Area of that piece is A sector  $2\pi - 2\pi \cdot \frac{2}{3} = 4\pi/3$ ...  $\frac{4\pi}{3} (\pi r^2) = \frac{2}{3} (9\pi) = 6\pi$  ← circle part...  $\int_{\frac{5\pi}{6}}^{2\pi} (4 - 2\sin\theta)^2 d\theta$

$$A = \frac{4\pi}{3} (\pi r^2) = \frac{2}{3} (9\pi) = 6\pi \leftarrow \text{circle part...} \int_{\frac{5\pi}{6}}^{2\pi} (4 - 2\sin\theta)^2 d\theta = 24.708$$

(9)

$$\textcircled{B} \quad r = 4 - 2\sin\theta \quad \theta = t^2$$

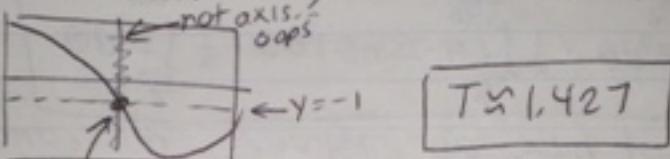
$$x = r \cos\theta$$

$$x = (4 - 2\sin\theta) \cos\theta$$

$$x = (4 - 2\sin(t^2)) \cos(t^2)$$

$$\underbrace{-1}_{y_1} = \underbrace{(4 - 2\sin(t^2)) \cos(t^2)}_{y_2 \text{ (use } x \text{ for } t)}$$

make  $x_{\min} = .9$ ,  $x_{\max} = 2.1$   $x_{\text{scale}} = 1$



$$T \approx 1.427$$

**2nd** **(CALC) INT**  
 $(1.427, -1)$   
 $T$

NOTE for C...  
 POSITION =  $\langle x(t), y(t) \rangle$   
 VECTOR

$$\textcircled{C} \quad x(t) = (4 - 2\sin(t^2)) \cos(t^2) \quad \leftarrow \text{already in as } y_2$$

$v'(t)$  isn't worth finding by hand...

they want  $x'(1.5)$ ...

you can do this in two ways in calc...

① **math** **8**  $\rightarrow$  **NDerive**( $y_2$ ,  $x$ , 1.5)  
 $\uparrow$   $\uparrow$   
 $x(t)$  standing  
 in for  $t$

or ② graph  $y_2 = (4 - 2\sin(x^2)) \cos(x^2)$   
**2nd** **(CALC)** **1b: dy/dx** then type in 1.5

(answer is displayed as  $dy/dx =$  —  
 at the bottom of the graph)

$$x'(1.5) \approx -8,072$$

## POLAR UNIT - 2013 Part C continued

We know  $x'(t) = -8.072 \dots$

$$y(t) = r \sin \theta = (4 - 2 \sin(t^2)) \sin(t^2)$$

Again, it's not worth finding

$y'(1.5)$  by hand --

if you use FXN mode...

$$Y_2 = (4 - 2 \sin(x^2)) \sin(x^2)$$

$$\boxed{\text{MATH} \quad 8 : n\text{Deru}(Y_2, X, 1.5) = -1.672 \stackrel{(3)}{=} Y'}$$

$$Y'(1.5) \approx -1.672 \stackrel{(3)}{=}$$

$$\text{so } v(1.5) = \langle x'(1.5), y'(1.5) \rangle$$

$$\boxed{v(1.5) = \langle -8.072, -1.672 \stackrel{(3)}{=} \rangle}$$

OR use Parametric mode

$$x(t) = (4 - 2 \sin(t^2)) \cos(t^2)$$

$$y(t) = (4 - 2 \sin(t^2)) \sin(t^2)$$

graph it (use zoom 6)

2ND CALC: 2:  $dy/dt$  and 3:  $dx/dt$

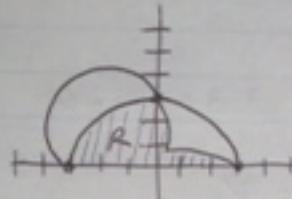
after you pick one of them enter the  
t-value (1.5) - then repeat for the  
other letter

$$\frac{dy}{dt} \approx -1.672 \stackrel{(3)}{=} \quad \frac{dx}{dt} \approx -8.072$$

$$\boxed{\text{ANS: } v(1.5) = \langle -8.072, -1.672 \stackrel{(3)}{=} \rangle}$$

## Polar UNIT - AP 2014 #2 (calculator)

- (2)  $r=3$  (circle)  $r=3-2\sin(2\theta)$  not circle...  
From  $0 \leq \theta \leq \pi$ .



NOTE: FOR  $[0, \frac{\pi}{2}]$  the area  
is inside the weird curve  
FOR  $[\frac{\pi}{2}, \pi]$  the area  
is inside the circle

(A)

so...  $A = \frac{1}{2} \int_0^{\frac{\pi}{2}} (3-2\sin(2\theta))^2 d\theta + \frac{1}{2} \int_{\frac{\pi}{2}}^{\pi} 3^2 d\theta$

you  
must have

integrands,  
constants,  
limits  
correct to  
get full  
AP credit

then use your calculator  
to evaluate

$$r_1 = 3 - 2\sin(2\theta)$$

$$A = .5 \text{ FNINT}(r_1^2, 0, 0, \frac{\pi}{2}) + .5 \text{ FNINT}(9, 0, \frac{\pi}{2}, \pi)$$

$$A \approx 9.707$$

(8)

(B)  $r = 3-2\sin(2\theta)$

$$x = r\cos\theta = (3-2\sin 2\theta) \cos \theta$$

$$\frac{dx}{d\theta} = (3-2\sin 2\theta)(-\sin \theta) + \cos \theta (-2\cos 2\theta \cdot 2)$$

$$\left. \frac{dx}{d\theta} \right|_{\frac{\pi}{6}} = (3-2\sin \frac{\pi}{3})(-\sin \frac{\pi}{6}) + (\cos \frac{\pi}{6})(-2\cos \frac{\pi}{3} \cdot 2)$$

$$= \left(3 - 2\frac{\sqrt{3}}{2}\right) \left(-\frac{1}{2}\right) + \frac{\sqrt{3}}{2} \left(-2\left(\frac{1}{2}\right) \cdot 2\right)$$

$$= (3 - \sqrt{3})(-\frac{1}{2}) + \frac{\sqrt{3}}{2}(-2)$$

$$= (3 - \sqrt{3})(-.5) + -\sqrt{3} = \boxed{-2.366}$$

oops.  
calc section.  
should have  
just typed

(C) Distance between two curves is  
just outer curve - inner curve  
between  $0 < \theta < \pi/2$  (aka 1st quadrant)  
the circle is outside

so

$$D = 3 - (3 - 2\sin 2\theta) = 3 - 3 + 2\sin 2\theta$$

$$D = 2\sin 2\theta$$

we want  $\frac{dD}{d\theta}$  when  $\theta = \pi/3 \dots$

$$\left. \frac{dD}{d\theta} \right|_{\pi/3} = \boxed{-2} \quad \leftarrow \text{you can just use your calc to evaluate by calling } D = 2\sin 2\theta$$

$$y_1 = 2\sin 2x$$

$$\text{DERIVE } (y_1, x, \pi/3) = -2$$

OR you can do it by hand (but why??)

$$\frac{dD}{d\theta} = 2(\cos 2\theta) \cdot 2 = 4\cos 2\theta$$

$$\left. \frac{dD}{d\theta} \right|_{\pi/3} = 4\cos^2 \frac{\pi}{3} = 4(-\frac{1}{2}) = -2$$

(D)  $r = 3 - 2\sin 2\theta \quad \frac{d\theta}{dt} = 3 \quad \text{FIND } \frac{dr}{dt} \text{ when } \theta = \pi/6$

$\downarrow \text{Derive w.r.t. } t \dots$

$$\frac{dr}{dt} = -2\cos 2\theta \cdot 2 \frac{d\theta}{dt} \Leftarrow -4\cos 2\theta \frac{d\theta}{dt}$$

$$\frac{dr}{dt} = -4\cos 2(\pi/6) \cdot 3 = -4\cos \frac{\pi}{3} \cdot 3$$

$$\frac{dr}{dt} = -4(\frac{1}{2}) \cdot 3 = -2 \cdot 3 = \boxed{-6}$$