

Integration by Substitution

Evaluate each indefinite integral. Use the provided substitution.

1) $\int -15x^4(-3x^5 - 1)^5 dx; u = -3x^5 - 1$

2) $\int -16x^3(-4x^4 - 1)^{-5} dx; u = -4x^4 - 1$

3) $\int -\frac{8x^3}{(-2x^4 + 5)^5} dx; u = -2x^4 + 5$

4) $\int (5x^4 + 5)^{\frac{2}{3}} \cdot 20x^3 dx; u = 5x^4 + 5$

5) $\int \frac{(5 + \ln x)^5}{x} dx; u = 5 + \ln x$

6) $\int 4 \sec 4x \cdot \tan 4x \cdot \sec^4 4x dx; u = \sec 4x$

7) $\int 36x^3(3x^4 + 3)^5 dx; u = 3x^4 + 3$

8) $\int x(4x - 1)^4 dx; u = 4x - 1$

Evaluate each indefinite integral.

9) $\int -9x^2(-3x^3 + 1)^3 dx$

10) $\int 12x^3(3x^4 + 4)^4 dx$

11) $\int -12x^2(-4x^3 + 2)^{-3} dx$

12) $\int (3x^5 - 3)^{\frac{3}{5}} \cdot 15x^4 dx$

13) $\int (-2x^4 - 4)^4 \cdot -32x^3 dx$

14) $\int (e^{4x} - 4)^{\frac{1}{5}} \cdot 8e^{4x} dx$

15) $\int x(4x + 5)^3 dx$

16) $\int 5x\sqrt{2x + 3} dx$

Substitution for Definite Integrals

Express each definite integral in terms of u , but do not evaluate.

1) $\int_{-1}^0 \frac{8x}{(4x^2 + 1)^2} dx; u = 4x^2 + 1$

2) $\int_0^1 -12x^2(4x^3 - 1)^3 dx; u = 4x^3 - 1$

3) $\int_{-1}^2 6x(x^2 - 1)^2 dx; u = x^2 - 1$

4) $\int_0^1 \frac{24x}{(4x^2 + 4)^2} dx; u = 4x^2 + 4$

Evaluate each definite integral.

5) $\int_{-3}^0 -\frac{8x}{(2x^2 + 3)^2} dx; u = 2x^2 + 3$

6) $\int_0^1 \frac{16x}{(4x^2 + 4)^2} dx; u = 4x^2 + 4$

7) $\int_{-1}^0 18x^2(3x^3 + 3)^2 dx; u = 3x^3 + 3$

8) $\int_0^1 -\frac{8x}{(4x^2 + 2)^2} dx; u = 4x^2 + 2$

Calculus u-substitution

Name: _____

Integrate each:

$$\int (3 - x)^{10} dx$$

$$\int \sqrt{7x + 9} dx$$

$$\int \frac{x^3}{(1 + x^4)^{1/3}} dx$$

$$\int e^{5x+2} dx$$

$$\int 4 \cos(3x) dx$$

$$\int \frac{\sin(\ln x)}{x} dx$$

$$\int \frac{3x + 6}{x^2 + 4x - 3} dx$$

$$\int x 3^{x^2+1} dx$$

$$\int \frac{3}{x \ln x} dx$$

$$\int \frac{\cos(5x)}{e^{\sin(5x)}} dx$$

$$\int_0^{\sqrt{\pi}} x \sin(x^2) dx$$

$$\int (2x + 5)(x^2 + 5x)^7 dx$$

u-Substitution - Classwork

When you take derivatives of more complex expressions, you frequently have to use the chain rule to differentiate. The integration equivalent of the chain rule is called *u-substitution*. *u-substitution* allows you integrate expressions which do not appear integratable.

1) $\int x(x^2 - 1)^5 dx$ Set up a $u =$ _____. Find $\frac{du}{dx} =$ _____. Solve for $du =$ _____

You need to manufacture your du in the original expression. So you will have to multiply by _____ on the inside and thus multiply by _____ on the outside. Now change everything to u .

Now integrate in terms of u .

Finally, change back to the variable x and add C .

2) $\int (3x-2)^4 dx$

3) $\int \sqrt{5x-2} dx$

4) $\int 4(6x-1)^{2/3} dx$

5) $\int x\sqrt{x^2-2} dx$

6) $\int x^2\sqrt{1-4x^3} dx$

7) $\int \frac{x}{\sqrt[3]{2x^2-1}} dx$

8) $\int x^{1/2}(x^{3/2}+2)^9 dx$

9) $\int (x+2)\sqrt{x^2+4x-3} dx$

10) $\int (x+2)\sqrt{x-4} \, dx$

11) $\int \frac{x-5}{\sqrt{x-6}} \, dx$

12) $\int \frac{x^2}{\sqrt{x+1}} \, dx$

13) $\int \cos 4x \, dx$

14) $\int 3\sin(1-3x) \, dx$

15) $\int \sin^3 x \cos x \, dx$

16) $\int \tan 10x \sec 10x \, dx$

17) $\int \tan^2 x \sec^2 x \, dx$

18) $\int \sin x \sqrt{\cos x} \, dx$

19) $\int \frac{\cos x}{\sqrt{1-\sin x}} \, dx$

u-Substitution - Homework

1. $\int \sqrt{x-2} \, dx$

2. $\int (2x+3)^{11} \, dx$

3. $\int \sqrt{5x-1} \, dx$

4. $\int \sqrt[3]{6x+1} \, dx$

5. $\int 5(3-4x)^{2/3} \, dx$

6. $\int \frac{dx}{(8x-1)^3}$

7. $\int x(x^2+2)^6 \, dx$

8. $\int 6x^2 \sqrt{3x^3-1} \, dx$

9. $\int \left(1 + \frac{1}{x}\right)^3 \left(\frac{1}{x^2}\right) dx$

10. $\int x^{1/3} \left(x^{4/3} + 9\right)^8 dx$

11. $\frac{2}{3} \int \sqrt{4 - \frac{3}{5}x} \, dx$

12. $\int (3x+15)\sqrt{x^2+10x+4} \, dx$

13. $\int (x+2)\sqrt{x-2} \, dx$

14. $\int \frac{x^2}{\sqrt{x-4}} \, dx$

15. $\int \sin 5x \, dx$

16. $\int \cos \frac{x}{2} \, dx$

17. $\int \frac{1}{3} \sec^2 8x \, dx$

18. $\int \sin 4x \cos 4x \, dx$

19. $\int \cos^3 x \sin x \, dx$

20. $\int \tan x \sec^2 x \, dx$

21. $\int \sqrt{\cos 6x} \sin 6x \, dx$

22. $\int \frac{\sin x}{(4 - \cos x)^3} \, dx$

Integration by Substitution Worksheet

Name: _____

Common Integral formulas to remember:

$$\int u^n du = \frac{u^{n+1}}{n+1} + C \quad \int \sin u du = -\cos u + C \quad \int \cos u du = \sin u + C \quad \int \csc^2 u du = -\cot u + C$$

$$\int \sec^2 u du = \tan u + C \quad \int \sec u \tan u du = \sec u + C \quad \int \csc u \cot u du = -\csc u + C$$

$$\int \tan u du = -\ln|\cos u| + C \text{ or } \ln|\sec u| + C \quad \int \cot u du = \ln|\sin u| + C \quad \int \frac{du}{u} = \ln|u| + C$$

$$\int e^{ku} du = \frac{e^{ku}}{k} + C \quad \int a^{ku} du = \frac{a^{ku}}{k \ln|a|} + C$$

Evaluate the Integrals:

1. $\int (x-1)^{243} dx$

2. $\int \sqrt{1-x} dx$

3. $\int \frac{1}{\sqrt{1-x}} dx$

4. $\int x\sqrt{2x^2-1} dx$

5. $\int (1+x^3)3x^2 dx$

6. $\int x(x^2+9)^{10} dx$

7. $\int \frac{x^2}{\sqrt{1+x^3}} dx$

8. $\int \frac{dt}{2\sqrt{1+t}}$

9. $\int 2xe^{x^2} dx$

10. $\int \frac{\sin x}{\cos^2 x} dx$

11. $\int \frac{dx}{x\sqrt{25x^2-2}}$

12. $\int \frac{dx}{\sqrt{1-4x^2}}$

Evaluate the integrals:

13.
$$\int_1^0 \frac{3}{3x-2} dx$$

14.
$$\int_0^{\pi/4} \tan x \sec^2 x dx$$

15.
$$\int_0^3 \frac{1}{x+1} dx$$

16.
$$\int_1^2 \frac{2 \ln x}{x} dx$$

17.
$$\int_1^4 \frac{e^{\sqrt{x}}}{2\sqrt{x}} dx$$

18.
$$\int_{\pi/6}^{\pi/2} \sin^2 x \cos x dx$$

19.
$$\int_0^{\pi/2} 2^{\cos x} \sin x dx$$

20.
$$\int_0^1 \frac{4x dx}{\sqrt{1-x^4}}$$
 Hint: use $u = x^2$

21.
$$\int_0^1 \frac{x}{1+x^4} dx$$

22.
$$\int_{\ln 4}^{\ln 7} \frac{e^x}{1+e^x} dx$$

Name: _____ Group Members: _____

Math 1132Q Worksheet: U Substitution

Date: _____

Objective: The objective of this worksheet is to get automatic in solving integrals with "u" substitution

1. For each of the integral use some form of U-substitution and solve the problems.
2. These problems are meant to be solved at home and questions regarding any of these problems should be asked in the discussion class.

1.

$$\int (\sin^5(x) + 3\sin^3(x) - \sin(x))\cos(x)dx$$

3.

$$\int \sin(x)\sec^8(x)dx$$

2.

$$\int x^2(x^3 + 1)^{40}dx$$

4.

$$\int \frac{e^{2x}}{e^{2x} + 1}dx$$

#12

5.

$$\int_1^{e^2} \frac{\ln(x)}{x} dx$$

7.

$$\int_0^{\pi/4} \frac{\sin(x)}{\cos^3(x)} dx$$

6.

$$\int_0^2 x^3 \sqrt{16-x^4} dx$$

8.

$$\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$$